Week 1: Assignment 1

The due date for this assignment has passed.

As per our records you have not submitted this assignment.

Due on 2021-09-10, 23:59 IST.

1. Let \( f(x) = x^2 + 3x - 4 \) be a function that satisfies the three conditions in the definition of a limit. Then

\( f(x) \) is always a metric space.

\( f(x) \) covers an open metric space but \( (f(x), \mathbb{R}) \) is not a metric space.

It is possible that neither \( (f(x), \mathbb{R}) \) nor \( (f(x), \mathbb{R}) \) is a metric space.

Both \( (f(x), \mathbb{R}) \) and \( (f(x), \mathbb{R}) \) are metric spaces.

No, the statement is incorrect.

Accrued points: 1

2. Let \( S \) be a metric space. Which of the statements are true?

\( S \) is always a metric space.

Both \( (S, \mathbb{R}) \) and \( (S, \mathbb{R}) \) are metric spaces.

\( S \) is not always a metric space.

\( (S, \mathbb{R}) \) is always a metric space.

No, the statement is incorrect.

Accrued points: 1

3. Let \( S \) be a metric space. Which of the statements are true?

\( S \) has a complete metric space.

\( S \) is not a complete metric space.

\( S \) is a complete metric space.

\( S \) is always a metric space.

No, the statement is incorrect.

Accrued points: 1

4. Let \( f(x) \) be a metric space. Mark the true statements

There always exists a continuous function \( f: X \to Y \) where \( Y \) is any other metric space.

If the metric on \( X \) is the discrete metric then any function \( f: X \to Y \) where \( Y \) is any metric space is continuous.

If \( Y = \mathbb{R} \) the discrete metric then no function \( f: X \to Y \) is continuous.

If \( Y = \mathbb{R} \) is continuous then \( Y = \mathbb{R} \) is a metric space then \( Y \) is not continuous if the metric on both \( X \) and \( Y \) are changed to some other equivalent metrics.

No, the statement is incorrect.

Accrued points: 1

5. Continuity of functions between metric spaces can be characterized using which of the following?

Open sets

Closed sets

Sequences

None of the above

No, the statement is incorrect.

Accrued points: 1

6. Which of the following about closed and open balls in a metric space are true? Here by closed ball we mean the set of the form

\[ B(r, x) = \{ y \in X \mid d(x, y) \leq r \} \]

- Any open set is a union of open balls
- Any closed set is a finite union of closed balls
- The closure of the union of all balls of radius \( r \) is the union of all balls of radius \( r \)

No, the statement is incorrect.

Accrued points: 1

7. In which of the following the metric or \( g \) satisfies the distance 1 property?

- Euclidean metric
- The discrete metric
- The uniform metric

No, the statement is incorrect.

Accrued points: 1

8. In which of the following the matrix or \( B \) does the sequence \( B^n \) converge to \( I \)?

- Euclidean metric
- The discrete metric
- The uniform metric

No, the statement is incorrect.

Accrued points: 1

9. Which of the following subspaces of \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \) cannot possibly be the range of this function?

\[ f(x) = x^2 + 3x - 4 \]

\[ \mathbb{R} \]

\[ \mathbb{R}^2 \]

\[ \mathbb{R}^3 \]

No, the statement is incorrect.

Accrued points: 1

10. Let \( f(x) \) be a metric space and consider the function \( g(y) = \begin{cases} 1 & \text{if } y = 0 \\ 0 & \text{otherwise} \end{cases} \) defined by

\[ g(y) = \begin{cases} 1 & \text{if } y = 0 \\ 0 & \text{otherwise} \end{cases} \]

\[ \lim_{y \to 0} g(y) = 0 \]

\[ \lim_{y \to 0} g(y) = 1 \]

No, the statement is incorrect.

Accrued points: 1

11. Let \( f(a) \) be a given point. When the following set \( S \) is not the open unit ball centered at \( a \) under this norm?

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

No, the statement is incorrect.

Accrued points: 1

12. Let \( f(a) \) be a given point. Which of the following set \( S \) is the open unit ball centered at \( a \) under this norm?

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

No, the statement is incorrect.

Accrued points: 1

13. Let \( f(a) \) be a given point. Which of the following set \( S \) is the unit circle centered at \( a \) under this norm?

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

No, the statement is incorrect.

Accrued points: 1

14. Let \( f(a) \) be a given point. Which of the following set \( S \) is the unit circle centered at \( a \) under this norm?

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

\[ \{ x \in \mathbb{R} \mid d(x, a) < 1 \} \]

No, the statement is incorrect.

Accrued points: 1