Assignment 8

The due date for submitting this assignment has passed.

As per our records you have not submitted this assignment.

Due on 2021-03-17, 23:59 IST.

1) If v is an eigenvector of a matrix A, the Rayleigh quotient $\frac{v^T Av}{v^T v}$ is: *1 point*

- the dominant eigenvalue of A corresponding to v
- the eigenvalue of A corresponding to v
- the smallest eigenvalue of A corresponding to v
- none of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
the eigenvalue of A corresponding to v

2) Let A be an $m \times n$ matrix whose entries are all 1. If λ and μ are the largest eigenvalues of $A^T A$ and AA^T respectively, then *1 point*

- $\lambda = \mu = 1$
- $\lambda = n, \mu = m$
- $\lambda = m, \mu = n$
- $\lambda = \mu = m \times n$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\lambda = \mu = m \times n$

3) Which of these is true? *1 point*

- Power method finds the smallest eigenvalue whereas the inverse power method finds the largest eigenvalue.
- Both power method and inverse power method find the largest eigenvalue.
- Both power method and inverse power method find the smallest eigenvalue.
- Power method finds the largest eigenvalue whereas the inverse power method finds the smallest eigenvalue.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Power method finds the largest eigenvalue whereas the inverse power method finds the smallest eigenvalue.

4) The solution of the boundary value problem $y'' - 7y' + 12y = 0, y(0) = 1, y(1) = 0$ is: *1 point*

- $-2e^{(4x)} + 3e^{(3x)}$
- $-\frac{e^{(4x)}}{e-1} + \frac{e^{(3x+1)}}{e-1}$
- $-2e^{(4x-4)} + 3e^{(3x-3)}$
- None of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
$-\frac{e^{(4x)}}{e-1} + \frac{e^{(3x+1)}}{e-1}$

5) Consider the logistic population growth model $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$ with $K = 2000, r = 0.03$ and initial population $P(0) = 5000$. Approximately in how many years will the population become half? *1 point*

- 20
- 30
- 37
- 40

No, the answer is incorrect.
Score: 0
Accepted Answers:
37

6) If we want to solve a Riccati differential equation in SageMath, then which option should be used with `desolve`? *1 point*

- `show_method=True`
- `contrib_ode=True`
- `contrib_ode=False`
- `show_method=False`

No, the answer is incorrect.
Score: 0
Accepted Answers:
`contrib_ode=True`

7) Which of the following is not an algorithm available with the SageMath routine `desolve`? *1 point*

- `maxima`
- `fricas`
- `GAP`
- None of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
GAP

8) Suppose we solve the ODE $y' = x - 2y, y(0) = 1$, using the SageMath command `pts = eulers_method(x-2*y,0,1,0.1,2,algorithm="none")`. Then `pts` *1 point*

- contains 20 points
- contains 21 points
- contains 22 points
- is a table of solutions

No, the answer is incorrect.
Score: 0
Accepted Answers:
contains 22 points

9) Which of the following SageMath syntax will plot only the solution curve of the ODE $\frac{dy}{dx} = x - y, y(0) = 1$ without frame? *1 point*

- `streamline_plot(x-y,(x,0,1),(y,0,1.2))`
- `streamline_plot(x-y,(x,0,1),(y,0,1.2), frame=False)`
- `streamline_plot(x-y,(x,0,1),(y,0,1.2),start_points=[[0,1]])`
- `streamline_plot(x-y,(x,0,1),(y,0,1.2),start_points=[[0,1]], frame=False)`

No, the answer is incorrect.
Score: 0
Accepted Answers:
`streamline_plot(x-y,(x,0,1),(y,0,1.2),start_points=[[0,1]], frame=False)`

10) Which of the following SageMath syntax can be used to plot the solution curve along with slope fields of the initial value ODE $\frac{dy}{dx} = 2x + y$, $y(0) = 1$ using the RK4 method? *1 point*

- `desolve_rk4(2*x+y),y,ics=[0,1],ivar=x,end_points=[-1,3], thickness=1,output='slope_field')`
- `desolve_rk4((2*x+y),y,ics=[0,1],ivar=x,end_points=[-1,3], thickness=1)`
- `desolve_rk4((2*x+y),y,ics=[0,1],ivar=x,end_points=[-1,3], thickness=1,output='plot')`
- None of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
`desolve_rk4((2*x+y),y,ics=[0,1],ivar=x,end_points=[-1,3], thickness=1,output='slope_field')`

11) Which of the following SageMath syntax can be used to solve the 3rd order initial value ODE *1 point*

$x'''(t) = t + 2x(t) + 3x''(t) = 0, \quad x(0) = 1, x'(0) = 2, x''(0) = 3$

```
t = var('t'); x = function('x')(t)
desolve(diff(x,t,3)==t+2*x+3*diff(x,t,2),x,ics=[0,1,2,3])
```

```
t = var('t')
x1 = function('x1')(t); x2 = function('x2')(t)
x3 = function('x3')(t)
de1 = diff(x1,t) == x2;
de2 = diff(x2,t) == x3;
de3 = diff(x3,t) ==t+2*x2+3*x3
sol = desolve_system([de1, de2,de3], [x1,x2,x3],ics=[0,1,2,3])
```

```
t = var('t')
x1 = function('x1')(t); x2 = function('x2')(t)
x3 = function('x3')(t)
de1 = diff(x1,t) == x1;
de2 = diff(x2,t) == x2;
de3 = diff(x3,t) ==t+2*x2+3*x3
sol = desolve([de1, de2,de3], [x1,x2,x3],ics=[0,1,2,3])
```

```
t = var('t')
x1 = function('x1')(t); x2 = function('x2')(t)
x3 = function('x3')(t)
de1 = diff(x1,t) == x1;
de2 = diff(x2,t) == x2;
de3 = diff(x3,t) ==t+2*x2+3*x3
sol = desolve_system([de1, de2,de3], x1,x2,x3,ics=[0,1,2,3])
```

No, the answer is incorrect.
Score: 0
Accepted Answers:
```
t = var('t')
x1 = function('x1')(t); x2 = function('x2')(t)
x3 = function('x3')(t)
de1 = diff(x1,t) == x2;
de2 = diff(x2,t) == x3;
de3 = diff(x3,t) ==t+2*x2+3*x3
sol = desolve_system([de1, de2,de3], [x1,x2,x3],ics=[0,1,2,3])
```

12) Consider the Lotka-Volterra predator-prey model *1 point*

$$\frac{dx}{dt} = \alpha x - \beta xy$$
$$\frac{dy}{dt} = \delta xy - \gamma y$$

with $\alpha = 2/3, \beta = 4/3, \gamma = \delta = 1/2$. A population model is said to be in equilibrium when neither of the population levels is changing. What are the values of x and y such that the given model is in equilibrium? (Such a point (x, y) is known as a fixed point of the model.)

- $x = 1, y = 1$
- $x = 2, y = 1$
- $x = 1, y = 1/2$
- $x = 1/2, y = 1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$x = 1, y = 1/2$

13) Consider the 2^{nd} order initial value ODE $\frac{d^2x}{dt^2} + x = t$, with $x(0) = 1, x'(0) = 2$. Let $X(s) = \mathcal{L}(x(t))$. If we solve this problem using the Laplace transform, then $X(s)$ is given by: *1 point*

- $\frac{s^3 + 2s^2 + 1}{s^4 + s^2}$
- $\frac{2s^2 + 1}{s^4 + s^2}$
- $\frac{1}{s^4 + s^2}$
- $\frac{1}{s^4 + s^2}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{s^3 + 2s^2 + 1}{s^4 + s^2}$

14) Consider the 2^{nd} order system of initial value ODEs *1 point*

$$\frac{d^2x}{dt^2} = y(t) + \sin t, \text{ with } x(0) = 1, x'(0) = 0.$$
$$\frac{d^2y}{dt^2} = -\frac{dx}{dt} + \cos t, \text{ with } y(0) = -1, y'(0) = -1.$$

Let $X(s) = \mathcal{L}(x(t))$ and $Y(s) = \mathcal{L}(y(t))$. If we solve this problem using the Laplace transform, then

- $X(s) = \frac{-(s+1)}{s^2+1}, Y(s) = \frac{s}{s^2+1}$
- $X(s) = \frac{s}{s^2+1}, Y(s) = \frac{-(s+1)}{s^2+1}$
- $X(s) = \frac{1}{s^2+1}, Y(s) = \frac{s}{s^2+1}$
- $X(s) = \frac{s}{s^2+1}, Y(s) = \frac{1}{s^2+1}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$X(s) = \frac{s}{s^2+1}, Y(s) = \frac{-(s+1)}{s^2+1}$

15) The function `unit_step(t-a)` is a unit step function in SageMath which is 0 if $t \leq 1$ and 1 if $t \geq 1$. Suppose $f(t)$ is a function which is 1 when $t \in [1, 2) \cup [3, 4]$ and 0 otherwise. Which of the following SageMath code can be used to represent $f(t)$? *1 point*

```
unit_step(t-1)+unit_step(t-2)+unit_step(t-3)+unit_step(t-4)
```

```
unit_step(t-1)-unit_step(t-2)-unit_step(t-3)-unit_step(t-4)
```

```
unit_step(t-1)-unit_step(t-2)+unit_step(t-3)-unit_step(t-4)
```

```
unit_step(t)-unit_step(t-1)+unit_step(t-2)-unit_step(t-2)
```

No, the answer is incorrect.
Score: 0
Accepted Answers:
`unit_step(t-1)-unit_step(t-2)+unit_step(t-3)-unit_step(t-4)`