Assignment 7

The due date for submitting this assignment has passed.

Due on 2021-03-10, 23:59 IST.

As per our records you have not submitted this assignment.

1) Let $A = \begin{pmatrix} -13 & -4 & -12 & -16 & 60 \\ 9 & 7 & -12 & -25 & -39 \\ 24 & 8 & -1 & -16 & -96 \\ -12 & -4 & 0 & 7 & 48 \\ -1 & 0 & -4 & -7 & 6 \end{pmatrix}$. Suppose A_4 is the matrix obtained by taking the sum of the first three terms in the rank one decomposition $A = \sum_i \sigma_i u_i v_i^T$ obtained using the singular value decomposition of A. Then the approximate value of $||A - A_4||$ is *1 point*

- 0.016285
- 0.075767
- 4.2021270484618944e-14
- 2.983948

No, the answer is incorrect.
Score: 0
Accepted Answers:
0.075767

2) Consider the matrix $B = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$. Let $A = U\Sigma V^T$ be the singular value decomposition of A where U and V are given by column matrices $U = [u_1\ u_2, u_3]$ and $V = [v_1\ v_2]$ respectively. Then the vector u_3 is *1 point*

- $(\frac{3}{14}\sqrt{7}\sqrt{2},\ \frac{1}{14}\sqrt{7}\sqrt{2},\ \frac{1}{7}\sqrt{7}\sqrt{2})$
- $(-\frac{1}{10}\sqrt{5}\sqrt{2},\ \frac{3}{10}\sqrt{5}\sqrt{2},\ 0)$
- $(-\frac{3}{35}\sqrt{7}\sqrt{5},\ -\frac{1}{35}\sqrt{7}\sqrt{5},\ \frac{1}{7}\sqrt{7}\sqrt{5})$
- $(\frac{1}{2},\ \sqrt{2},\ \frac{1}{2}\sqrt{2})$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(-\frac{3}{35}\sqrt{7}\sqrt{5},\ -\frac{1}{35}\sqrt{7}\sqrt{5},\ \frac{1}{7}\sqrt{7}\sqrt{5})$

3) Consider the system of ODE's *1 point*

$$x_1'(t) = 2x_1 + x_2 + x_3$$
$$x_2'(t) = 2x_1 + x_2 - 2x_3$$
$$x_3'(t) = -x_1 + 2x_3$$

with $x_1(0) = -1, x_2(0) = 2, x_3(0) = 3$. Then $x_1(t) + x_2(t) + x_3(t)$ is

- $4e^t$
- $3e^t$
- e^t
- 0

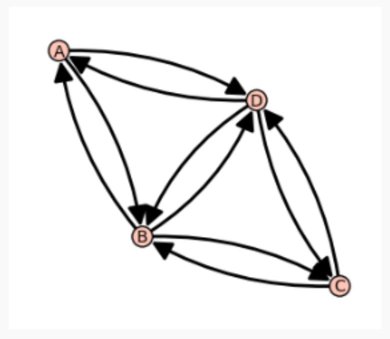
No, the answer is incorrect.
Score: 0
Accepted Answers:
$4e^t$

4) Let $A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 2 \end{pmatrix}$. Then the Jordan canonical form of A is given by *1 point*

- $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- None of the these

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

5) Consider the network *1 point*

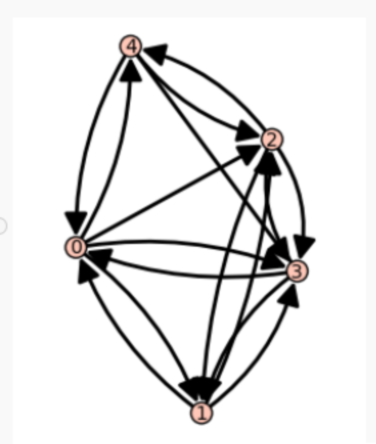


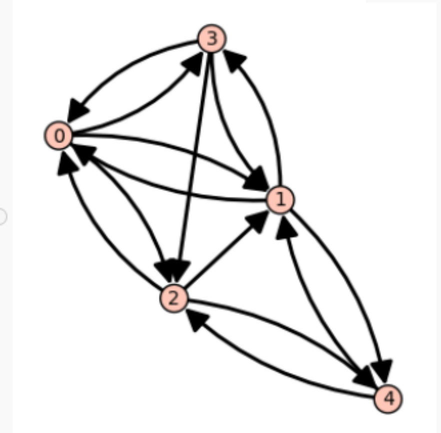
The adjacency matrix of the above network is given by

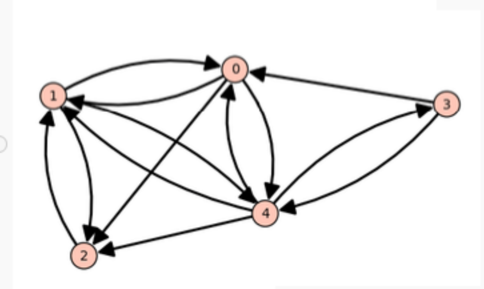
- $\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$
- $\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$
- $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$
- None of these

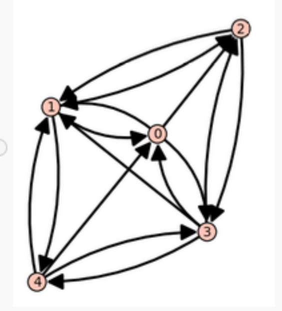
No, the answer is incorrect.
Score: 0
Accepted Answers:
$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

6) Which of the following networks has the adjacency matrix $\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$? *1 point*

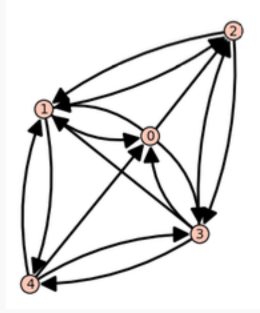








No, the answer is incorrect.
Score: 0
Accepted Answers:



7) Consider the function $f(x) = -2\,x^3 + \cos(x)^2 + xe^{(-x)}$. The Sage command `f.find_root()`, if used to find a root in the interval $[-1, 1]$ with `xtol=1e-10`, uses: *1 point*

- 8 function calls and 9 iterations to find a root.
- 8 function calls and 8 iterations to find a root.
- 9 function calls and 8 iterations to find a root.
- 9 function calls and 9 iterations to find a root.

No, the answer is incorrect.
Score: 0
Accepted Answers:
9 function calls and 8 iterations to find a root.

8) If the Gauss-Jacobi iteration scheme to solve $AX = B$ is $X_{k+1} = HX_k + C$, then *1 point*

- $H = -(L + D)^{-1}U$ and $C = (L + D)^{-1}B$.
- $H = -(L + D)^{-1}U$ and $C = D^{-1}B$.
- $H = -D^{-1}(L + U)$ and $C = D^{-1}B$.
- $H = -D^{-1}(L + U)$ and $C = (L + D)^{-1}B$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$H = -D^{-1}(L + U)$ *and* $C = D^{-1}B$.

9) Consider the successive over relaxation (SOR) scheme *1 point*

$$X_{k+1} = HX_k + C$$

to solve $AX = B$ where

$$H = (D + \omega L)^{-1}\left[(1 - \omega)D - \omega U\right] \quad \text{and} \quad C = \omega(D + \omega L)^{-1}B.$$

Then this scheme is convergent

- for any starting vector $x^{(0)}$.
- for any starting vector $x^{(0)}$ if spectral radius of H is less than 1.
- for any starting vector $x^{(0)}$ if spectral radius of H is greater than 1.
- for any starting vector $x^{(0)}$ if and only if A is symmetric and positive definite.

No, the answer is incorrect.
Score: 0
Accepted Answers:
for any starting vector $x^{(0)}$ *if and only if* A *is symmetric and positive definite.*

10) If $f(x)$ is the Lagrange interpolating polynomial passing through the set of points $(1, 2), (3, 2), (4, 5), (7, 4), (9, 2)$, then $f(10)$ will output to *1 point*

- 2.5
- an error
- 9.34999999999993
- 0

No, the answer is incorrect.
Score: 0
Accepted Answers:
9.34999999999993

11) If S is the cubic spline through the set of points $(1, 2), (3, 2), (4, 5), (7, 4,)$ and $(9, 2)$, then the value of S(-1), S(4.5), S(10) returns *1 point*

- (nan, 5.902210202991453, nan)
- (nan, nan, nan)
- (nan, 5.0, nan)
- None of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
(nan, 5.902210202991453, nan)

12) If $f(x) = x^2, g(x) = x^3$ and $h(x) = x^4$ and I_f, I_g and I_h are exact integrals of f, g and h respectively in the interval [0,1]. Suppose I_1, I_2 and I_3 are numerical integrals of f, g and h obtained using the Simpson's 1/3-rd composite formula with $n = 10$ in the interval [0,1] . Supposing that we use $a \approx b$ to represent a is approximately equal to b, which of the following options is correct? *1 point*

- $I_f = I_1, I_g = I_2, I_h = I_3$.
- $I_f \neq I_1, I_g \neq I_2, I_h \neq I_3$.
- $I_f \approx I_1, I_g \approx I_2, I_h \approx I_3$.
- $I_f = I_1, I_g = I_2, I_h \approx I_3$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$I_f = I_1, I_g = I_2, I_h \approx I_3$.

13) Use the python command *1 point*

```
from scipy.integrate import simps, trapz.
```

Let $f(x) = x/(1 + x^2)$. Suppose I_1 and I_2 are values of the integral of f obtained using `trapz` and `simps` functions with $n = 12$ in the interval [0,1] . Let I be the exact integral of f, $e_1 = |I - I_1|$ and $e_2 = |I - I_2|$. Then which one is the most appropriate?

- $e_1 = e_2$
- $e_1 < e_2$
- $e_1 > e_2$
- None of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
$e_1 > e_2$

14) Consider the system of linear equations $Ax = b$ where $A = \begin{pmatrix} 10.0 & 7.0 & -2.0 \\ 3.0 & -10.0 & 2.0 \\ 2.0 & 10.0 & 20.0 \end{pmatrix}$ and $b = \begin{pmatrix} 1.0 \\ 1.0 \\ -1.0 \end{pmatrix}$. If H is the iteration matrix of the Gauss-Sieldel iteration scheme $X_{k+1} = HX_k + C$ then what is the spectral radius of H? *1 point*

- 1
- -0.3954065922853803
- 0.03540659228538013
- 0.3954065922853803

No, the answer is incorrect.
Score: 0
Accepted Answers:
0.3954065922853803

15) Which of the following is not true about the Newton-Raphson method? *1 point*

- It has higher rate of convergence than the bisection method.
- It has higher rate of convergence than the secant method.
- It is a two point iteration scheme.
- It is a single point iteration scheme.

No, the answer is incorrect.
Score: 0
Accepted Answers:
It is a two point iteration scheme.