Course outline



# Assignment 6

**The due date for submitting this assignment has passed.**

**Due on 2021-03-03, 23:59 IST.**

**As per our records you have not submitted this assignment.**

1) Let $V$ be a finite dimensional vector space over $\mathbb{R}$, and $B_1$ and $B_2$ be two ordered bases of $V$. Let $T: V \to V$ be a linear transformation. Suppose $A$ and $B$ are matrices of $T$ with respect to bases $B_1$ and $B_2$ respectively. Which of the following is **not** correct? *1 point*

- $A$ and $B$ have the same eigenvalues.
- $A$ and $T$ have the same eigenvalues.
- Eigenvalues of $A$ and $B$ may not be same.
- $B$ and $T$ have the same eigenvalues.

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
*Eigenvalues of $A$ and $B$ may not be same.*

2) Let $A$ be a matrix whose entries are defined over a field $X$ in SageMath. With which of the following commands in SageMath, can we change the field $X$ to a field $Y$? *1 point*

- `A.change_ring(X,Y)`
- `A.change_ring(Y)`
- `A.change_field(Y)`
- `change_ring(A,Y)`

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
`A.change_ring(Y)`

3) Suppose we use the SageMath command *1 point*

```
A = random_matrix(QQ, 5, algorithm='diagonalizable',
eigenvalues=[-1,1,2], dimensions=[1,2,2])
```

to generate a matrix $A$. Then the multiplicities of eigenvalues 2, 1 and -1 are

- 1, 2, 2 respectively.
- 2, 1, 2 respectively.
- 2, 2, 1 respectively.
- None of these

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
*2, 2, 1 respectively.*

4) Suppose we define *1 point*

```
A1 = matrix(QQ,4,[-13, 48, 36, -48, 0, -1, 0, 0,
-2, 8, 5, -8, 2, -8, -6, 7])
A2 = matrix(RR,4,[-13, 48, 36, -48, 0, -1, 0, 0,
-2, 8, 5, -8, 2, -8, -6, 7])
```

Then using the SageMath commands

```
A1.diagonalization()
A2.diagonalization()
```

we can check that

- both A1 and A2 are diagonalizable.
- neither A1 nor A2 is diagonalizable.
- A1 is not diagonalizable but A2 is diagonalizable.
- A1 is diagonalizable but A2 is not diagonalizable.

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
*A1 is diagonalizable but A2 is not diagonalizable.*

5) Consider the matrix $A = \begin{pmatrix} 0.40 & 0.20 & 0.30 \\ 0.30 & 0.60 & 0.19 \\ 0.25 & 0.15 & 0.50 \end{pmatrix}$. Then $A^k$ as $k \to \infty$ is *1 point*

- a $3 \times 3$ matrix all of whose entries are $\infty$.
- a $3 \times 3$ matrix all of whose entries are 1.
- a zero matrix of order $3 \times 3$
- None of these

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
*a zero matrix of order $3 \times 3$*

6) Consider an inner product defined on $M_n(\mathbb{R})$ as *1 point*

$$\langle A, B \rangle := \text{trace}(AB^T).$$

What is the norm of the matrix

$$A = \begin{pmatrix} -8 & 1 & -3 & 1 & 7 \\ -3 & 1 & 0 & -1 & 19 \\ 2 & 1 & -12 & -89 & 0 \\ -2 & 2 & 0 & 2 & -11 \\ -1 & 1 & 0 & -1 & -2 \end{pmatrix}$$

upto 2 decimal places with respect to the above inner product?

- 86.89
- 89.87
- 93.31
- 73.65

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
*93.31*

7) In SageMath define an inner product on $C[0, 1]$ as *1 point*

$$\langle f, g \rangle := \int_0^1 f(x)g(x)\,dx.$$

Then the orthogonal projection of $x^2 - x + 6$ onto $x^3 - 20x^2 - 3$ with respect to this inner product is (coefficients rounded off up to 2 decimal places)?

- $-0.46\,x^3 + 9.10\,x^2 + 7.36$
- $-0.45\,x^3 + 2.10\,x^2 + 1.36$
- $-0.45\,x^3 + 9.10\,x^2 + 1.36$
- $-0.46\,x^3 + 2.10\,x^2 + 7.38$

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
$-0.45\,x^3 + 9.10\,x^2 + 1.36$

8) Let $P: \mathbb{R}^2 \to \mathbb{R}^2$ and $Q: \mathbb{R}^2 \to \mathbb{R}^2$ denote the reflection and orthogonal projection about the line $y = mx$ respectively. Let $A$ and $B$ be the matrices of $P$ and $Q$ with respect to the standard bases. Then *1 point*

- $A = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$ and $B = \frac{1}{1+m^2}\begin{bmatrix} 1 & m \\ m & m^2 \end{bmatrix}$
- $A = \frac{1}{1+m^2}\begin{bmatrix} 1 & m \\ m & m^2 \end{bmatrix}$ and $B = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$
- $A = B = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$
- $A = B = \frac{1}{1+m^2}\begin{bmatrix} 1 & m \\ m & m^2 \end{bmatrix}$

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
$A = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$ *and* $B = \frac{1}{1+m^2}\begin{bmatrix} 1 & m \\ m & m^2 \end{bmatrix}$

9) Consider an inner product on $\mathbb{R}^4$ defined as $\langle u, v \rangle = v^T A u$ where *1 point*

$$A = \begin{pmatrix} 22 & 10 & -8 & -1 \\ 10 & 7 & -1 & 2 \\ -8 & -1 & 6 & 4 \\ -1 & 2 & 4 & 27 \end{pmatrix}.$$

Then the orthonormal basis of $\mathbb{R}^4$ using vectors

$$\{(1, 3, -3, 0), (3, 0, 2, 2), (-2, 7, 3, 2), (-2, 12, 0, 1)\}$$

with respect to the above inner product obtained by the Gram-Schmidt process is? (Use .n(4) to get 4 digits for each basis vector obtained.)

- $\{(1.0, 3.0, -3.0, 0.00)(2.5, -1.8, 3.8, 2.0)(-5.5, 10., -2.8, -1.0)(1.6, -2.0, 2.0, -0.12)\}$
- $\{(0.37, 0.12, -0.12, 0.), (-0.59, 0.35, -0.15, 0.29), (-0.88, 0.29, 0.13, 0.88), (0.12, -0.25, 0.58, 0.58)\}$
- $\{(0.062, 0.19, -0.19, 0.00), (0.19, -0.14, 0.28, 0.16), (-0.50, 0.94, -0.25, -0.094), (1.8, -2.0, 2.0, -0.12)\}$
- $\{(0.27, 0.22, -0.12, 0.), (-0.51, 0.35, -0.15, 0.22), (-0.88, 0.29, 0.13, 0.88), (0.12, 0.25, 0.58, 0.28)\}$

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
$\{(0.062, 0.19, -0.19, 0.00), (0.19, -0.14, 0.28, 0.16), (-0.50, 0.94, -0.25, -0.094), (1.8, -2.0, 2.0, -0.12)\}$

10) Consider the matrix *0 points*

$$A = \begin{pmatrix} 3 & 1 & 2 & 6 & -4 \\ 1 & 0 & 0 & 3 & -2 \\ -1 & -1 & -3 & 1 & 0 \\ 0 & 0 & -5 & 5 & 0 \\ 0 & -2 & -4 & 6 & -4 \end{pmatrix}$$

defined over a $QQ$ field in SageMath. Then which of the following is non zero?

- `((A.right_kernel()).random_element()*(A.row_space()).random_element())`
- `((A.kernel()).random_element()*(A.column_space()).random_element())`
- `((A.T.right_kernel()).random_element()*(A.T.row_space()).random_element())`
- All of these

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
*All of these*

11) Let $W$ be the subspace of $\mathbb{R}^4$ with basis $\{(-2, 12, 0, 1), (1, 7, 5, 4), (4, 3, -1, 2), (-4, 19, 3, 3)\}$. Then the projection matrix to find the projection $p$ from vector $v$ onto the subspace $W$ is: *1 point*

- $\begin{pmatrix} 0 & 0 & 2 & 1 \\ 4 & 3 & 2 & 0 \\ 1 & 2 & 0 & 4 \\ 0 & 1 & 1 & 0 \end{pmatrix}$
- $\begin{pmatrix} 1 & 4 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 1 & 3 & 0 & -44 \end{pmatrix}$
- $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- $\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

12) The least square solution of the system of linear equations *1 point*

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 10 \\ x_1 + 2x_2 - x_3 &= 7 \\ 3x_1 + 4x_2 + 2x_3 &= 3 \\ -3x_1 + 2x_2 + x_3 &= 5 \end{aligned}$$

- Does not exist
- Is equal to $\begin{pmatrix} 3/71 \\ 221/355 \\ 723/355 \end{pmatrix}$
- Is equal to $\begin{pmatrix} 723/355 \\ 3/71 \\ 221/355 \end{pmatrix}$
- Is equal to $\begin{pmatrix} 221/355 \\ 723/355 \\ 3/71 \end{pmatrix}$

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
*Is equal to* $\begin{pmatrix} 3/71 \\ 221/355 \\ 723/355 \end{pmatrix}$

13) The best fit cubic polynomial through the points *1 point*

$$\{(1, 2), (3, -8), (8, 3), (10, 4), (11, 9), (14, 18), (7, 4), (2, 2), (15, 16)\}$$

using the least square fitting (Round off up-to 4 decimal places) is:

- $-0.0291\,x^3 + 0.8421\,x^2 - 5.4348\,x + 6.6106$
- $-0.0111\,x^3 + 0.3132\,x^2 - 1.73\,x + 1.7215$
- $-0.0252\,x^3 + 0.7532\,x^2 - 4.8812\,x + 5.9083$
- $0.0167\,x^3 - 0.2529\,x^2 + 1.5529\,x - 2.4241$

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
$-0.0252\,x^3 + 0.7532\,x^2 - 4.8812\,x + 5.9083$

14) Let $A$ be a random matrix defined over the field RDF in SageMath. Suppose `X,Y,Z = A.SVD()`. Then which of the following is not true? *1 point*

- $X$ and $Z$ are orthogonal matrices
- $A = X * Y * Z^T$
- $A = X * Y * Z$
- None of these

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
$A = X * Y * Z$

15) Let $A$ be a random matrix defined over the field RDF in SageMath. Which of the following SageMath commands can find the singular values of $A$? *1 point*

- `[sqrt(z) for z in (A*A.T).eigenvalues()]`
- `[sqrt(z) for z in (A.T*A).eigenvalues()]`
- `A.singular_values()`
- All of these

No, the answer is incorrect.
Score: 0
**Accepted Answers:**
*All of these*