# Assignment 5

**The due date for submitting this assignment has passed.**

**Due on 2021-02-24, 23:59 IST.**

**As per our records you have not submitted this assignment.**

1) Consider the problem *1 point*

optimize $f(x, y) = x^2 + 2y^2 - x$, subjected to $g(x, y) = x^2 + y^2 - 1 = 0$.

If we solve this problem using the method of Lagrange multipliers, which of the following is a point of minimum of $f$ ?

- $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
- $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- $(1, 0)$
- $(-1, 0)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(1, 0)$

2) Consider the problem *1 point*

optimize $f(x, y, z) = x^2 + y^2 - z$

subject to $g_1(x, y, z) = x^2 + y^2 - z^2 = 0$, $g_2(x, y, z) = x^2 + z^2 = 9$.

If we solve this problem using the method of Lagrange multipliers, then the maximum value of $f(x, y, z)$ is:

- 2.37
- 6.62
- 16.25
- 12.00

No, the answer is incorrect.
Score: 0
Accepted Answers:
12.00

3) Suppose we declare a vector in SageMath as `v = vector([3,4,5])`. Then `v` can be equivalently represented as: *1 point*

- vector(RR,[3,4,5])
- vector(ZZ,[3,4,5])
- vector([3.0,4.0,5.0])
- vector(EC,[3,4,5])

No, the answer is incorrect.
Score: 0
Accepted Answers:
vector(ZZ,[3,4,5])

4) Which of the following gives the echelon matrix of any matrix $A$? *1 point*

- `A.echelon_form()`
- `A.echelonize()`
- `A.rref()`
- All of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
All of these

5) Let $c1$, $c2$ and $c3$ be three vectors in the 3 dimensional space. Suppose we declare these vectors in SageMath and use the command `P = v1.plot()+v2.plot()+v3.plot()` for plotting them together. Then which of the following will plot the graph? *1 point*

- `show(P)`
- `display(P)`
- `P`
- All of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
All of these

6) Let $p$ be a list and we declare following SageMath commands *1 point*

```
vec = vector(p)
sup_norm = vec.norm(Infinity)
taxicab_norm = vec.norm(1)
euclidean_norm = vec.norm(2)
```

For which choice of list $p$, are both the following criteria satisfied?

$a$ : The `sup_norm` of vector(p) divides its `taxicab_norm`.
$b$ : The `Euclidean_norm` of vector(p) is not a multiple of `sup_norm`.

- [1,4,4]
- [4,4,2]
- [2,4,4,4,2]
- [4,4,4,4]

No, the answer is incorrect.
Score: 0
Accepted Answers:
[2,4,4,4,2]

7) Suppose the scalar triple product of three vectors $p$, $q$ and $r$ is 42 and the dot product of $p$ and $q$ is 24. Then which of the following can be the vectors $p$, $q$ and $r$ respectively? *1 point*

- $[2, 4, 5]$, $[5, 4, 2]$, $[2, 4, 3]$
- $[1, 2, 3]$, $[6, 4, 1]$, $[2, 6, 5]$
- $[1, 2, 4]$, $[6, 5, 2]$, $[5, 6, 4]$
- $[1, 2, 5]$, $[2, 1, 4]$, $[5, 6, 3]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[1, 2, 5]$, $[2, 1, 4]$, $[5, 6, 3]$

8) For any matrix $A$ , the SageMath command `A.add_multiple_of_row(i,j,a)` : *1 point*

- adds $a$ times $(j + 1)^{th}$ row to $(i + 1)^{th}$ row
- adds $a$ times $j^{th}$ row to $i^{th}$ row
- adds $a$ times $i^{th}$ row to $j^{th}$ row
- add $a$ times $(i + 1)^{th}$ row to $(j + 1)^{th}$ row

No, the answer is incorrect.
Score: 0
Accepted Answers:
adds $a$ times $(j + 1)^{th}$ row to $(i + 1)^{th}$ row

9) Let $W$ be the subspace of the vector space $V = \mathbb{Q}^3$ over the rational field with a basis $B = \{(1, 3, 4), (2, -2, 1), (1, -3, -1)\}$. The coordinates of the vector $(-7, 1, 3)$ with respect to basis $B$ of $W$ are *1 point*

- $[17/20, -11/4, -47/20]$
- $[22, -38, 47]$
- $[-36/7, 76/7, -89/7]$
- $[19, -12, 34]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[22, -38, 47]$

10) Let $B_1 = \{(-1, -1, -1, 0, -1, 0), (-1, 0, -9, -4, 1, 1), (1, 1, 1, 502, -9, -4), (-2, -1, 0, 2, 1, -1), (0, 1, -1, -9, -2, -4)\}$ and $B_2 = \{(0, 0, -3, 0, 3, -1), (-1, 0, 71, 0, -1, 6), (-1, 60, 2, -2, 1, 0), (5, -4, -9, -7, 0, -109), (1, -2, -6, -1, 0, 1)\}$. If $W_1 = L(B_1)$ and $W_2 = L(B_2)$ are linear spans of $B_1$ and $B_2$ respectively, then the dimensions of $W_1 + W_2$ and $W_1 \cap W_2$ in order are *1 point*

- 6 and 4
- 7 and 3
- 3 and 7
- 5 and 5

No, the answer is incorrect.
Score: 0
Accepted Answers:
6 and 4

11) Let $S$ be a vector space over finite field defined in SageMath. Which of the following line gives a random element of $S$ and the number of elements in $S$ respectively? *1 point*

- `S.an_element(),dim(S)`
- `S.random_element(),S.cardinality()`
- `S.an_element(),S.cardinality()`
- `S.random_element(),dim(S)`

No, the answer is incorrect.
Score: 0
Accepted Answers:
`S.random_element(),S.cardinality()`

12) For any matrix $A$, the rank of $A$ and nullity of $A^T$ are obtained in SageMath by: *1 point*

- `rank(A),dim(A.T).right_kernel())`
- `dim(A.row_space()),dim(A.left_kernel())`
- `dim(A.column_space()),dim(kernel(A))`
- All of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
All of these

13) Which of the following proves the Rank Nullity theorem for matrix $A$ given by *1 point*

$$A = \begin{pmatrix} 3 & -3 & 5 & 7 & -3 & -1 & 3 \\ 0 & 0 & 1 & 4 & 6 & 2 & 6 & 16 \\ 3 & -2 & 9 & 13 & -1 & 5 & 19 \\ 3 & -4 & 1 & 1 & -5 & -7 & -13 \\ 3 & -1 & 13 & 19 & 1 & 11 & 35 \end{pmatrix}$$

- `dimension(A.row_space())+dimension(A.right_kernel())==A.nrows()`
- `dimension(A.column_space())+dimension(A.left_kernel())==A.ncols()`
- `dimension(A.row_space())+dimension(A.T.left_kernel())==A.nrows()`
- `dimension(A.column_space())+dimension(A.right_kernel())==A.ncols()`

No, the answer is incorrect.
Score: 0
Accepted Answers:
`dimension(A.column_space())+dimension(A.right_kernel())==A.ncols()`

14) Consider a linear transformation $T\colon \mathbb{Q}^3 \to \mathbb{Q}^4$ defined by *1 point*

$$T(x, y, z) = (x, x + 2z, 2x + y + 9z, x - 3y + 5z).$$

Which of the following is the matrix of $T$ with respect to basis

$$B_1 = \{(11, -6, 0), (0, 11, 0), (-1, 14, 3)\}$$

on the domain and the standard basis on the codomain?

- $\begin{pmatrix} 11 & 0 & -1 \\ 11 & 0 & 5 \\ 16 & 11 & 39 \\ 29 & -33 & -28 \end{pmatrix}$
- $\begin{pmatrix} 11 & 11 & 16 & 29 \\ 0 & 0 & 11 & -33 \\ -1 & 5 & 39 & -28 \end{pmatrix}$
- $\begin{pmatrix} 11 & 1 & 16 & 9 \\ 0 & 0 & 11 & -33 \\ -1 & 15 & 39 & 123 \end{pmatrix}$
- $\begin{pmatrix} 1 & 11 & 16 & 29 \\ 0 & 0 & 11 & -3 \\ -1 & & 39 & 8 \end{pmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\begin{pmatrix} 11 & 0 & -1 \\ 11 & 0 & 5 \\ 16 & 11 & 39 \\ 29 & -33 & -28 \end{pmatrix}$

15) Consider linear transformations $T_1\colon \mathbb{Q}^4 \to \mathbb{Q}^3$ defined by *1 point*

$$T_1(x, y, z, w) = (x + y - z + w, x + 4y - 5z - 2w)$$

and $T_2\colon \mathbb{Q}^3 \to \mathbb{Q}^4$ defined by

$$T_2(x, y, z) = (x, x + 2z, 2x + y + 9z, x - 3y + 5z)$$

Then the matrix of $T_1 \circ T_2$ with respect to the standard bases on domain and codomain is

- $\begin{pmatrix} 1 & 2 & -7 \\ -4 & 0 & 1 \\ -2 & 2 & -47 \end{pmatrix}$
- $\begin{pmatrix} 1 & -4 & -2 \\ 2 & 0 & 2 \\ -7 & 1 & -47 \end{pmatrix}$
- $\begin{pmatrix} 1 & 3 & 12 & 3 \\ 1 & 9 & 39 & 18 \\ -1 & -11 & -47 & -26 \\ 1 & -3 & -16 & -9 \end{pmatrix}$
- $\begin{pmatrix} 1 & 1 & -1 & 1 \\ 3 & 9 & -11 & -3 \\ 12 & 39 & -47 & -16 \\ 3 & 18 & -26 & -9 \end{pmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 3 & 9 & -11 & -3 \\ 12 & 39 & -47 & -16 \\ 3 & 18 & -26 & -9 \end{pmatrix}$