Assignment 5

Due on 2021-10-26, 11:59 PM

1. Solve the following system of equations.

\[
\begin{align*}
2x + 3y &= 11 \\
3x - 2y &= 4
\end{align*}
\]

2. Consider the function \( f(x) = x^3 - 3x + 2 \).

(a) Find the critical points of \( f(x) \).

(b) Determine the intervals where \( f(x) \) is increasing and decreasing.

3. Evaluate the limit \( \lim_{x \to 0} \frac{\sin x}{x} \).

4. Find the derivative of the function \( g(x) = 5x^4 - 2x^3 + 6x - 7 \).

5. Solve the following differential equation.

\[
\frac{dy}{dx} = y - x
\]

6. Given the function \( f(x) = \sqrt{x^2 + 1} \), find the derivative \( f'(x) \).

7. If \( a \) and \( b \) are constants, solve the following equation.

\[
\frac{1}{a} + \frac{1}{b} = \frac{2}{a + b}
\]

8. Solve for \( x \) in the equation \( 3^x = 81 \).

9. Given the equation \( f(x) = x^2 - 5x + 6 \), determine the values of \( x \) for which \( f(x) = 0 \).

10. A tank contains 100 gallons of a mixture of water and salt. Fresh water is added to the tank at a rate of 5 gallons per minute, and the mixture is drained from the tank at a rate of 4 gallons per minute. The concentration of salt in the tank is 10 pounds per gallon initially. Find the amount of salt in the tank after 1 hour.

\[
\frac{dS}{dt} = \text{rate of salt entering} - \text{rate of salt leaving}
\]

11. Find the indefinite integral of \( f(x) = e^{-x^2} \).

12. Evaluate the definite integral \( \int_0^1 x^2 \, dx \).

13. Given the function \( f(x) = \sin(x) \), find the second derivative \( f''(x) \).

14. Solve the differential equation \( y' = 2x \).

15. Find the Taylor series expansion of \( f(x) = e^x \) around \( x = 0 \) up to the second term.

16. Evaluate the limit \( \lim_{x \to 0} \frac{\sin(x) - x}{x^3} \).

17. Find the area under the curve \( y = x^2 + 1 \) from \( x = 0 \) to \( x = 2 \).

18. Evaluate the integral \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \).

19. Solve the differential equation \( y'' - 4y = 0 \).

20. Find the limit \( \lim_{x \to 0} \frac{\sin(x)}{x} \).

21. Evaluate the integral \( \int_0^\infty e^{-x} \, dx \).

22. Find the area under the curve \( y = \sqrt{1 - x^2} \) from \( x = -1 \) to \( x = 1 \).

23. Evaluate the limit \( \lim_{x \to 0} \frac{\cos(x) - 1}{x^2} \).

24. Find the area between the curves \( y = x^2 \) and \( y = 2x - x^2 \) from \( x = 0 \) to \( x = 2 \).

25. Solve the differential equation \( y'' + 4y = 0 \).

26. Evaluate the limit \( \lim_{x \to \infty} \frac{x^2}{e^x} \).

27. Find the area under the curve \( y = e^x \) from \( x = 0 \) to \( x = 1 \).

28. Evaluate the integral \( \int_0^1 \frac{1}{x} \, dx \).

29. Solve the differential equation \( y' = \frac{1}{x} \).

30. Evaluate the integral \( \int_0^\infty e^{-x^2} \, dx \).

31. Find the area under the curve \( y = \sqrt{x} \) from \( x = 0 \) to \( x = 4 \).

32. Evaluate the limit \( \lim_{x \to 0} \frac{\sin(2x)}{x^2} \).

33. Find the area under the curve \( y = x^3 \) from \( x = 0 \) to \( x = 1 \).

34. Evaluate the integral \( \int_0^\infty e^{-x} \, dx \).

35. Find the area under the curve \( y = \ln(x) \) from \( x = 1 \) to \( x = e \).

36. Evaluate the limit \( \lim_{x \to 0} \frac{\sin(3x)}{x^3} \).

37. Find the area under the curve \( y = e^{-x} \) from \( x = 0 \) to \( x = 1 \).

38. Evaluate the integral \( \int_0^\infty e^{-x^2} \, dx \).

39. Find the area under the curve \( y = x^2 \) from \( x = 0 \) to \( x = 2 \).

40. Evaluate the limit \( \lim_{x \to 0} \frac{\sin(4x)}{x^2} \).

41. Find the area under the curve \( y = \sqrt{x} \) from \( x = 0 \) to \( x = 4 \).

42. Evaluate the integral \( \int_0^\infty e^{-x^2} \, dx \).

43. Find the area under the curve \( y = x^3 \) from \( x = 0 \) to \( x = 1 \).

44. Evaluate the limit \( \lim_{x \to 0} \frac{\sin(2x)}{x} \).

45. Find the area under the curve \( y = e^x \) from \( x = 0 \) to \( x = 1 \).

46. Evaluate the integral \( \int_0^\infty e^{-x^2} \, dx \).

47. Find the area under the curve \( y = x^2 \) from \( x = 0 \) to \( x = 2 \).

48. Evaluate the limit \( \lim_{x \to 0} \frac{\sin(3x)}{x} \).

49. Find the area under the curve \( y = \ln(x) \) from \( x = 1 \) to \( x = e \).

50. Evaluate the integral \( \int_0^\infty e^{-x} \, dx \).