Assignment 7

The due date for submitting this assignment has passed.

Due on 2021-03-10, 23:59 IST.

1) Let \( r \) be a positive integer. Let \( F \) be a field such that the characteristic of \( F \) is either 0 or a prime number that doesn't divide \( r \). Let \( K \) be the splitting field of \( x^{n} - 1 \) over \( F \). Let \( (Z/nZ)^{r} \) denote the multiplicative group of units modulo \( n \). Select all the statements which are always true.

- \( K/F \) is Galois.
- \( K/F \) is cyclic.
- \( K/F \) is abelian.

No, the answer is incorrect.
Score: 0
Accepted Answer: \( K/F \) is Galois.
\( K/F \) is abelian.

2) Let \( K \) be the splitting field of \( x^{15} - 1 \) over \( Q \). Select all the correct statements.

- \( \left[ K : Q \right] = 6 \).

Let \( G \) be the Galois group of \( K/Q \). Then for every \( n \in G \), we have \( n^{15} = 1 \).

\( G \) contains an element of order 3.

There exists an intermediate field \( F \) for the extension \( K/Q \) such that \( K \) is a cyclic extension of \( F \) and \( \left[ K : F \right] = 4 \).

No, the answer is incorrect.
Score: 0
Accepted Answer: \( \left[ K : Q \right] = 6 \).
There exists an intermediate field \( F \) for the extension \( K/Q \) such that \( K \) is a cyclic extension of \( F \) and \( \left[ K : F \right] = 4 \).

3) Select all the correct statements.

- Any Galois extension \( K/Q \) of degree 4 is radical.
- Any Galois extension \( K/Q \) of degree 4 is simple radical.

Let \( F \) be a field of characteristic 0 and let \( K/F \) be a Galois extension of degree 6. Then every element of \( K \) is solvable over \( F \).

Every abelian extension is simple radical.

No, the answer is incorrect.
Score: 0
Accepted Answer: Any Galois extension \( K/Q \) of degree 4 is radical.
Let \( F \) be a field of characteristic 0 and let \( K/F \) be a Galois extension of degree 6. Then every element of \( K \) is solvable over \( F \).

4) Let \( f \) be a polynomial of characteristic 0. Let \( K \) be the splitting field of \( f \) over \( Q \). Assume that \( \alpha \) is the degree of \( f \) and \( G \) is the Galois group of \( K/F \). Select all the correct statements.

- \( g \in F[x] \) if \( a, \beta \) are roots of \( g \in K \), then there exists an element \( \sigma \in G \) such that \( \sigma(a) = \beta \).
- \( g \in F[x] \) of \( F[x] \) is irreducible, \( a, \beta \) are roots of \( g \in K \), then there exists an element \( \sigma \in G \) such that \( \sigma(a) = \beta \).
- \( G \) is contained in the alternating group \( A_{\alpha} \).
- If the discriminant of \( f \) is a square in \( F \), then \( G \) is contained in the alternating group \( A_{\alpha} \).

No, the answer is incorrect.
Score: 0
Accepted Answer: \( g \in F[x] \) if \( a, \beta \) are roots of \( g \in K \), then there exists an element \( \sigma \in G \) such that \( \sigma(a) = \beta \).
If the discriminant of \( f \) is a square in \( F \), then \( G \) is contained in the alternating group \( A_{\alpha} \).

5) Select all the correct statements.

- If \( G \) is a solvable group then \( G \) is abelian.

- The symmetric group \( S_{\alpha} \) is solvable for every integer \( \alpha \).
Every group of order at most 10 is solvable.

Let \( f \in Q[x] \) be a polynomial of degree at most 4. Then \( f \) is solvable over \( Q \).

No, the answer is incorrect.
Score: 0
Accepted Answer: Every group of order at most 10 is solvable.
Let \( f \in Q[x] \) be a polynomial of degree at most 4. Then \( f \) is solvable over \( Q \).

6) Let \( f \in Q[x] \) be an irreducible quartic polynomial over \( Q \) of characteristic 0 field \( F \). Let \( D \in F \), \( g \in F[x] \) denote the discriminant and the resultant cubic of \( f \), respectively. Let \( K \) be the splitting field of \( f \) over \( F \). Select all the correct statements.

- If \( f \) splits completely in \( F \), then \( \left[ K : F \right] = 2 \).
- If \( f \) splits completely in \( F \), then \( \left[ K : F \right] = 4 \).
- If \( \left[ K : F \right] = 12 \) then \( D \) is not a square in \( F \).

If \( K/F \) is a cyclic extension then \( D \) is not a square in \( F \).

No, the answer is incorrect.
Score: 0
Accepted Answer: If \( f \) splits completely in \( F \), then \( \left[ K : F \right] = 4 \).
If \( K/F \) is a cyclic extension then \( D \) is not a square in \( F \).