Assignment 2

The due date for submitting this assignment has passed.

As per our records you have not submitted this assignment.

1) Let \( n \) be a positive integer and let \( C_n \) denote a cyclic group of order \( n \). Let \( Q, R \) denote the fields of rational numbers and real numbers, respectively. Select all the correct statements.

- The degree of \( C_n \) over \( Q \) is \( n \) for every positive integer \( n \).
- The degree of \( C_n \) over \( Q \) is \( p - 1 \) for every prime number \( p \).
- If \( C_n \subseteq R \) then \( n = 1 \) or \( n = 2 \).
- If \( C_n \) belongs to a field \( K \), then \( C_n \) also belongs to \( K \).

No, the answer is incorrect. Score: 0.

Accepted Answers:
- The degree of \( C_n \) over \( Q \) is \( p - 1 \) for every prime number \( p \).
- If \( C_n \subseteq R \) then \( n = 1 \) or \( n = 2 \).

2) Select all the correct statements. Let \( C \) denote the field of complex numbers. Let \( Z \) denote the ring of integers.

- If \( G \) is a cyclic group of order 10, then there are exactly 10 characters of \( G \) in \( C^* \).
- The symmetric group \( S_5 \) has exactly two characters in \( C^* \).
- The cyclic group \( Z/6Z \) has exactly 2 characters in \( C^* \).
- The Klein 4-group has exactly one character in \( C^* \).

No, the answer is incorrect. Score: 0.

Accepted Answers:
- If \( G \) is a cyclic group of order 10, then there are exactly 10 characters of \( G \) in \( C^* \).
- The symmetric group \( S_5 \) has exactly two characters in \( C^* \).

3) Let \( G \) denote the field of rational numbers. Let \( K = Q(\sqrt{2}, i) \) and let \( G \) denote the group of all field homomorphisms \( K \rightarrow K \). Here \( i \) denotes a 4th root of \(-1\). Select all the correct statements.

- If \( \sigma \in G \) then \( \sigma \) is a \( Q \)-automorphism of \( K \).
- The fixed field of \( G \) is \( K \).
- Let \( H \) be a proper subgroup of \( G \). Then the fixed field of \( H \) is \( Q \).
- The fixed field of \( G \) is \( Q \).

No, the answer is incorrect. Score: 0.

Accepted Answers:
- If \( \sigma \in G \) then \( \sigma \) is a \( Q \)-automorphism of \( K \).
- The fixed field of \( G \) is \( Q \).

4) Let \( G = Q(\sqrt{2}) \) and let \( G \) denote the group of all field homomorphisms \( K \rightarrow K \). Select all the correct statements.

- The fixed field of \( G \) is \( Q \).
- The fixed field of \( G \) is \( K \).
- The fixed field of \( G \) is an intermediate field \( L \) such that \( |L : Q| = 2 \).

No, the answer is incorrect. Score: 0.

Accepted Answers:
- The fixed field of \( G \) is an intermediate field \( L \) such that \( |L : Q| = 2 \).
- There is a subgroup \( M \) of \( G \) such that the fixed field of \( M \) is \( K \).

5) Let \( K = F_p \) be the field of order \( p^2 \) and let \( \Phi : K \rightarrow K \) be the Frobenius homomorphism. So \( \Phi(x) = x^p \) for every \( x \in K \). Let \( G \) be the group of automorphisms of \( K \) generated by \( \Phi \). Select all the correct statements.

- The order of \( G \) is \( 2 \).
- The order of \( G \) is \( 3 \).
- The fixed field of \( G \) is \( F_p^2 \).

No, the answer is incorrect. Score: 0.

Accepted Answers:
- The order of \( G \) is \( 2 \).
- There is a subgroup \( M \) of \( G \) such that the fixed field of \( M \) is \( F_p^2 \).

6) Let \( G \) be a field with prime field \( F \). Let \( G \) be a group of automorphisms of \( K \). Let \( K^G \) denote the fixed field of \( G \). Select all the correct statements.

- \( F \subseteq K^G \).
- \( K : K^G = |G| \).
- \( K^G = \{ x \in K : \Phi(x) = x \} \).
- \( K^G \subseteq K \).

No, the answer is incorrect. Score: 0.

Accepted Answers:
- \( F \subseteq K^G \).
- \( K : K^G = |G| \).