Assignment 9

Due on 2020-11-18, 23:59 IST.

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

1) Which of the following functions defined on [0, 1] are integrable? 

   • The function \( x^2 \)
   • The Dirichlet function
   • The function \( \sqrt{x} \)
   • The topologist's sine curve.

   No, the answer is incorrect.
   Score 0
   Accepted Answers:
   The function \( x^2 \)
   The function \( \sqrt{x} \)

2) Let \( f \) be an increasing and bounded function on \( [a, b] \rightarrow \mathbb{R} \). Then

   • \( f \) is integrable.
   • \( f \) is integrable if \( f \) is also continuous.
   • \( f \) is integrable if \( f \) is strictly increasing and need not be integrable if \( f \) is merely increasing.
   • Irrespective of the partition \( P \) of \([a, b] \), we always have \( L(f, P) \leq U(f, P) \).

   No, the answer is incorrect.
   Score 0
   Accepted Answers:
   \( f \) is integrable.

3) Let \( f : [0, 1] \rightarrow \mathbb{R} \) be integrable. Then

   • If we modify the value of \( f \) at one point then the modified function is also integrable.
   • If we modify the value of \( f \) at finitely many points then the modified function is also integrable.
   • If the value of \( f \) is modified on a set of measure zero then the modified function is integrable.
   • If we modify the value of \( f \) at countably many points then the modified function is also integrable.

   No, the answer is incorrect.
   Score 0
   Accepted Answers:
   If we modify the value of \( f \) at one point then the modified function is also integrable.
   If we modify the value of \( f \) at finitely many points then the modified function is also integrable.
   If the value of \( f \) is modified on a set of measure zero then the modified function is integrable.
   If we modify the value of \( f \) at countably many points then the modified function is also integrable.

4) Which of the following properties about a function \( f : [a, b] \rightarrow \mathbb{R}, a < b \) is guaranteed if it is integrable?

   • \( f \) is differentiable at least at one point in \([a, b]\)
   • \( f \) is continuous at least at one point in \([a, b]\)
   • \( f \) satisfies the intermediate value property
   • The set of discontinuities of \( f \) cannot be uncountable

   No, the answer is incorrect.
   Score 0
   Accepted Answers:
   \( f \) is continuous at least at one point in \([a, b]\)

5) Let \( f, g : [a, b] \rightarrow \mathbb{R} \) be integrable. Then

   • The function \( |f| \) is integrable.
   • The function \( f^2 \) is integrable.
   • The function \( \min\{f^2, g^2\} \) is integrable.
   • The function \( \max\{f^2, g^2\} \) is integrable.

   No, the answer is incorrect.
   Score 0
   Accepted Answers:
   The function \( |f| \) is integrable.
   The function \( f^2 \) is integrable.
   The function \( \min\{f^2, g^2\} \) is integrable.
   The function \( \max\{f^2, g^2\} \) is integrable.

6) Which of the following were used in the proof of the Riemann-Lebesgue theorem?

   • The fundamental theorem of calculus.
   • Existence of Lebesgue number for an open cover of a compact set.
   • Countable union of sets of measure zero is a set of measure zero.
   • The notion of the oscillation of a function at a point.

   No, the answer is incorrect.
   Score 0
   Accepted Answers:
   Existence of Lebesgue number for an open cover of a compact set.
   The countable union of sets of measure zero is a set of measure zero.
   The notion of the oscillation of a function at a point.

7) Which of the following were used to prove that any continuous function \( f : [a, b] \rightarrow \mathbb{R} \) is integrable?

   • Heine-Borel theorem.
   • Uniform continuity.
   • The minimum value theorem
   • Countable union of sets of measure zero is a set of measure zero.

   No, the answer is incorrect.
   Score 0
   Accepted Answers:
   Uniform continuity.
   The minimum value theorem.