Unit 9 - Week 7

Course outline

Assignment 7

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-04, 23:59 IST.

1) Which of the following facts are used to prove the intermediate value theorem for a continuous function $f:[a,b]\to\mathbb{R}$. *0 points*

- Connectedness of $[a,b]$.
- Compactness of $[a,b]$
- Boundedness of $[a,b]$
- The fact that image of a connected set under a continuous map is connected.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Connectedness of $[a,b]$.

2) Which of the following are true about connected sets? *1 point*

- Arbitrary unions of connected sets are connected.
- Arbitrary intersections of connected sets are connected.
- The only subsets of the Cantor set that are connected are the empty set and singleton sets.
- The complement of a connected set is never connected.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Arbitrary intersections of connected sets are connected.
The only subsets of the Cantor set that are connected are the empty set and singleton sets.

3) Which of the following sets are perfect? *1 point*

- The closed interval [0, 1].
- The open interval (0, 1).
- The Cantor set.
- The set $\mathbb{R}\setminus\mathbb{N}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The closed interval [0, 1].
The Cantor set.

4) Which of the following facts were used to prove that the set of discontinuities of a function $f:\mathbb{R}\to\mathbb{R}$ is a F_σ -set. *1 point*

- The notion of oscillation of a function at a point.
- Uniform continuity.
- The Baire category theorem.
- The extreme value theorem.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The notion of oscillation of a function at a point.

5) Let $f:[a,b]\to\mathbb{R}$ be a continuous map. Let $B:=f([a,b])$. Then B is *1 point*

- A closed interval.
- An interval but not necessary closed.
- A compact set.
- A connected set.

No, the answer is incorrect.
Score: 0
Accepted Answers:
A closed interval.
A compact set.
A connected set.

6) Which of the following statements are true? *1 point*

- If $A\subset\mathbb{R}$ is connected then so is $\overline{A}$.
- If $A\subset\mathbb{R}$ is compact then so is $\overline{A}$.
- If $A\subset\mathbb{R}$ is perfect then so is $\overline{A}$.
- If $A\subset\mathbb{R}$ is not connected then $\overline{A}$ is also not connected.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $A\subset\mathbb{R}$ is connected then so is $\overline{A}$
If $A\subset\mathbb{R}$ is compact then so is $\overline{A}$
If $A\subset\mathbb{R}$ is perfect then so is $\overline{A}$

7) Let $\{x_1,x_2,\ldots\}$ be an enumeration of $\mathbb{Q}$. Let *0 points*

$$V:=\bigcup_n B\left(x_n,\frac{1}{2^n}\right)$$

Which of the following are true statements.

- $V=\mathbb{R}$
- V is an open set that contains $\mathbb{Q}$ and $\mathbb{R}\setminus V$ is a closed set that contains only irrational numbers.
- The set $\mathbb{R}\setminus V$ is perfect.
- $\mathbb{R}\setminus V$ does not contain any non-empty interval.

No, the answer is incorrect.
Score: 0
Accepted Answers:
V is an open set that contains $\mathbb{Q}$ and $\mathbb{R}\setminus V$ is a closed set that contains only irrational numbers.
The set $\mathbb{R}\setminus V$ is perfect.

8) Which of the following statements are true? *1 point*

- The complement of a G_δ set is a F_σ set.
- The set of rational numbers is a G_δ set.
- A countable intersection of G_δ sets is a G_δ set.
- The set $\mathbb{Q}$ is an F_σ set and the set $\mathbb{R}\setminus\mathbb{Q}$ is a G_δ set.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The complement of a G_δ set is a F_σ set.
A countable intersection of G_δ sets is a G_δ set.
The set $\mathbb{Q}$ is an F_σ set and the set $\mathbb{R}\setminus\mathbb{Q}$ is a G_δ set.

9) Which of the following sets are nowhere dense? *1 point*

- $\mathbb{Q}$
- The Cantor set.
- Any countable set.
- Any uncountable set whose complement is also an uncountable set.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The Cantor set.

10) Which of the following requests are possible? *1 point*

- A continuous function $f:(0,1)\to\mathbb{R}$ and Cauchy sequence $x_n\in(0,1)$ with $f(x_n)$ not a Cauchy sequence.
- A continuous function $f:[0,1]\to\mathbb{R}$ and Cauchy sequence $x_n\in(0,1)$ with $f(x_n)$ not a Cauchy sequence.
- A continuous function $f:(0,1)\to\mathbb{R}$ that attains its maximum value but not its minimum value.
- A continuous function $f:[0,\infty)\to\mathbb{R}$ that takes a sequence x_n diverging to ∞ to a sequence that converges to 0.

No, the answer is incorrect.
Score: 0
Accepted Answers:
A continuous function $f:(0,1)\to\mathbb{R}$ and Cauchy sequence $x_n\in(0,1)$ with $f(x_n)$ not a Cauchy sequence.
A continuous function $f:(0,1)\to\mathbb{R}$ that attains its maximum value but not its minimum value.
A continuous function $f:[0,\infty)\to\mathbb{R}$ that takes a sequence x_n diverging to ∞ to a sequence that converges to 0.