Assignment 6
Due on 2020-10-28, 23:59:59

The following problems ask you to show that certain sets are compact. As always, a set is compact if and only if it is closed and bounded.

1. Let $f: \mathbb{R} \to \mathbb{R}$ be a function. Let $A \subseteq \mathbb{R}$ be a subset of the real numbers. Let $f(A)$ be the set of all $y \in \mathbb{R}$ such that $x \in A$ implies $f(x) \leq y$. Let $g: \mathbb{R} \to \mathbb{R}$ be another function. Let $g(A)$ be the set of all $y \in \mathbb{R}$ such that $x \in A$ implies $g(x) \leq y$. Prove that if $f$ is continuous and $g$ is continuous, then $f(A)$ and $g(A)$ are both closed sets.

2. Let $f: \mathbb{R} \to \mathbb{R}$ be a function. Let $A \subseteq \mathbb{R}$ be a subset of the real numbers. Let $f(A)$ be the set of all $y \in \mathbb{R}$ such that $x \in A$ implies $f(x) \leq y$. Let $g: \mathbb{R} \to \mathbb{R}$ be another function. Let $g(A)$ be the set of all $y \in \mathbb{R}$ such that $x \in A$ implies $g(x) \leq y$. Prove that if $f$ is continuous and $g$ is continuous, then $f(A)$ and $g(A)$ are both closed sets.

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9. Let $f: \mathbb{R} \to \mathbb{R}$ be a function. Let $A \subseteq \mathbb{R}$ be a subset of the real numbers. Let $f(A)$ be the set of all $y \in \mathbb{R}$ such that $x \in A$ implies $f(x) \leq y$. Let $g: \mathbb{R} \to \mathbb{R}$ be another function. Let $g(A)$ be the set of all $y \in \mathbb{R}$ such that $x \in A$ implies $g(x) \leq y$. Prove that if $f$ is continuous and $g$ is continuous, then $f(A)$ and $g(A)$ are both closed sets.

10. Let $f: \mathbb{R} \to \mathbb{R}$ be a function. Let $A \subseteq \mathbb{R}$ be a subset of the real numbers. Let $f(A)$ be the set of all $y \in \mathbb{R}$ such that $x \in A$ implies $f(x) \leq y$. Let $g: \mathbb{R} \to \mathbb{R}$ be another function. Let $g(A)$ be the set of all $y \in \mathbb{R}$ such that $x \in A$ implies $g(x) \leq y$. Prove that if $f$ is continuous and $g$ is continuous, then $f(A)$ and $g(A)$ are both closed sets.