Assignment 5

Due: Fall 2020-10-21, 23:59 EST

The text below is the content of the assignment. Please read it carefully and attempt the exercises.

1. List the three statements:
   a. Some open set is open.
   b. The complement of an open set can never be open.
   c. Any subset of \( \mathbb{R} \) is either open or closed.

2. Which of the following is true about \( \mathbb{Q} \):
   a. Each point of \( \mathbb{Q} \) is a limit point of \( \mathbb{Q} \).
   b. Every limit point of \( \mathbb{Q} \) is in \( \mathbb{Q} \).
   c. Every isolated point of \( \mathbb{Q} \) is also a limit point.

3. Suppose your teacher is to prove that \( A \subseteq \mathbb{R} \) is open. Which of the following are plausible strategies?
   a. Show every point of \( A \) is an interior point of \( A \).
   b. Show every point of \( A \) is an isolated point of \( A \).
   c. Show every point of \( A \) is an interior point of \( A \).

4. Let \( \mathbb{Q} \) be the set of all rational numbers.
   a. \( \mathbb{Q} \) is not open.
   b. \( \mathbb{Q} \) is not closed.
   c. \( \mathbb{Q} \) is not empty.

5. For \( f, g : \mathbb{R} \rightarrow \mathbb{R} \), which of the following is possible?
   a. Both \( f \) and \( g \) are continuous at every point of \( \mathbb{R} \).
   b. \( f \) is not continuous at \( 0 \) but \( g \) is.
   c. \( f \) is not continuous at \( 0 \) and \( g \) is continuous at \( 0 \).

6. Consider the absolute value function \( f(x) = |x| \), defined on \( \mathbb{R} \). Then:
   a. \( f \) is continuous on the rational numbers of \( \mathbb{R} \).
   b. The function \( f \) is continuous on the whole of \( \mathbb{R} \).
   c. The function \( f \) is not continuous at \( 0 \).

7. Suppose you are given a function \( f : \mathbb{R} \rightarrow \mathbb{R} \) and are asked to show that \( f \) is not continuous at \( 0 \). Which of the following is a plausible strategy?
   a. Show that the sequence \( (a_n) \) does not converge to \( 0 \).
   b. Show that \( (a_n) \) is finite and that \( f(a_n) \) does not converge to \( f(0) \).
   c. Show that \( (a_n) \) is infinite and \( f(a_n) \) does not converge to \( f(0) \).

8. Consider the function \( f(x) = \frac{1}{x} \) for \( x \neq 0 \). Then which of the following are continuous at \( x = 0 \)?
   a. \( f \) is continuous at \( x = 0 \).
   b. \( f \) is not continuous at \( x = 0 \).
   c. \( f \) is continuous at \( x = 0 \).

9. Which of the following identifies about closure and interior are true?
   a. \( A \) is an open set if and only if its closure equals itself.
   b. \( A \) is a closed set if and only if it contains its boundary.
   c. \( A \) is neither open nor closed.

10. Which of the following pairs are possible?
    a. A countable subset of \( \mathbb{R} \) and an uncountable subset of \( \mathbb{R} \).
    b. A countable subset of \( \mathbb{R} \) and an uncountable subset of \( \mathbb{R} \).
    c. A measurable subset of \( \mathbb{R} \) and a non-measurable subset of \( \mathbb{R} \).

11. Which of the following remains possible?
    a. A countable subset of \( \mathbb{R} \) and a non-measurable subset of \( \mathbb{R} \).
    b. A countable subset of \( \mathbb{R} \) and a measurable subset of \( \mathbb{R} \).
    c. A non-measurable subset of \( \mathbb{R} \) and a measurable subset of \( \mathbb{R} \).