Unit 14 - Week 12

Assignment 12

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-12-09, 23:59 IST.

Note: In this assignment, $f:[a,b]\to\mathbb{R}$ can be uniformly approximated by polynomials means that we can find a sequence P_n that converges uniformly to f.

1) Consider the function $\sin:\mathbb{R}\to\mathbb{R}$. Which of the following statements are true? *1 point*

- sin cannot be uniformly approximated by polynomials in $(0,2\pi)$ because the Weierstrass approximation theorem requires the interval of definition to be closed.
- sin can be uniformly approximated by polynomials in $(0,2\pi)$ using the Weierstrass approximation theorem.
- sin can be uniformly approximated by polynomials on any interval of finite length.
- We can uniformly approximate sin by polynomials on any finite length inteval by using the Taylor series of sin.

No, the answer is incorrect.
Score: 0
Accepted Answers:
sin can be uniformly approximated by polynomials in $(0,2\pi)$ using the Weierstrass approximation theorem.
sin can be uniformly approximated by polynomials on any interval of finite length.
We can uniformly approximate sin by polynomials on any finite length inteval by using the Taylor series of sin.

2) Let $P_n:[a,b]\to\mathbb{R}$ be a sequence of polynomials of fixed degree $k\geq 1$ converging pointwise to a function $f:[a,b]\to\mathbb{R}$ *1 point*

- Then f is continuous.
- f is a polynomial.
- P_n converges uniformly and not just pointwise.
- f is differentiable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Then f is continuous.
f is a polynomial.
P_n converges uniformly and not just pointwise.
f is differentiable.

3) Suppose we we know that $f,g:[a,b]\to\mathbb{R}$ are two functions that can be uniformly approximated by polynomials. Which of the following functions can then also be uniformly approximated by polynomials without using the Weierstrass approximation theorem. *1 point*

- $f+g$.
- $max\{f,g\}$.
- kf where $k\in\mathbb{R}$.
- If the absolute value function $|\cdot|:[a,b]\to\mathbb{R}$ can also uniformly approximated by polynomials then $max\{f,g\}$ can be uniformly approximated by polynomials.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f+g$.
kf where $k\in\mathbb{R}$.
If the absolute value function $|\cdot|:[a,b]\to\mathbb{R}$ can also uniformly approximated by polynomials then $max\{f,g\}$ can be uniformly approximated by polynomials.

4) Let $f:[a,b]\to\mathbb{R}$ be a function that is continuously differentiable. Then *1 point*

- f' can be uniformly approximated by polynomials.
- f can be uniformly approximated by polynomials.
- We can find a sequence of polynomials P_n such that P_n converges uniformly to f on $[a,b]$ and P_n' converges uniformly to f' on $[a,b]$.
- If P_n is a sequence of polynomials converging uniformly to f then P_n' converges uniformly to f'.

No, the answer is incorrect.
Score: 0
Accepted Answers:
f' can be uniformly approximated by polynomials.
f can be uniformly approximated by polynomials.
We can find a sequence of polynomials P_n such that P_n converges uniformly to f on $[a,b]$ and P_n' converges uniformly to f' on $[a,b]$.

5) Suppose you want to find a sequence of polynomials P_n that converges **pointwise** to a continuous function $f:(a,b)\to\mathbb{R}$, where a and b are real numbers with $a<b$. Which of the following combined allows us to achieve this. *1 point*

- We can apply the Weierstrass approximation theorem on any closed subset of (a,b).
- We can find a sequence of closed sets $K_n\subset(a,b)$ such that
$$\bigcup_n K_n=(a,b)$$
- It is not always possible to find such a sequence P_n
- We can apply the Weierstrass approximation theorem on $[a,b]$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
We can apply the Weierstrass approximation theorem on any closed subset of (a,b).
We can find a sequence of closed sets $K_n\subset(a,b)$ such that
$$\bigcup_n K_n=(a,b)$$