Unit 3 - Week 1

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Assignment 1

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-09-30, 23:59 IST.

More than one answer may be right. Partial marks are awarded if only some of the correct answers are selected. No marks awarded if even one of the wrong answers is selected.

1) Mark the options that are correct: *1 point*

- A statement is a sentence that is true.
- A statement and its negation may both be false.
- A statement and its negation may both be true.
- To prove that a statment is true, it is enough to show that the negation is false.

No, the answer is incorrect.
Score: 0

Accepted Answers:
To prove that a statment is true, it is enough to show that the negation is false.

2) Which of the following English statements are appropriate ways of stating that $p \implies q$: *1 point*

- q follows from p.
- p is a necessary condition for q.
- q is a necessary condition for p.
- p is a sufficient conditon for q.

No, the answer is incorrect.
Score: 0

Accepted Answers:
q follows from p.
q is a necessary condition for p.
p is a sufficient conditon for q.

3) Which of the following statements are true: *1 point*

- If 4 is even then 7 is odd.
- If 8 is even and 2 is not prime, then 3 > 5.
- It is not the case that 5 is even and 7 is prime.
- Any natural number which is not prime must either be even or odd.

No, the answer is incorrect.
Score: 0

Accepted Answers:
If 4 is even then 7 is odd.
If 8 is even and 2 is not prime, then 3 > 5.
It is not the case that 5 is even and 7 is prime.
Any natural number which is not prime must either be even or odd.

4) Which of the following relationships are possible between three sets A, B and C: *1 point*

- $A \in B$ and $A \subset B$
- $A \in B$ and $B \in A$
- $A \in B, B \in C$ and $C \in A$
- There is a set D such that such that $D \subset A, B, C$ but $A \cap B \cap C = \emptyset$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$A \in B$ and $A \subset B$
There is a set D such that such that $D \subset A, B, C$ but $A \cap B \cap C = \emptyset$

5) Which of the following is true *1 point*

- To prove $\forall n \ \ p(n)$ is true, it takes only one example.
- To prove $\exists n \ \ p(n)$ is true, it takes only one example.
- To prove $\forall n \neg p(n)$ is true, it takes only one example.
- You can never prove a theorem by just exhibiting one example.

No, the answer is incorrect.
Score: 0

Accepted Answers:
To prove $\exists n \ \ p(n)$ is true, it takes only one example.

6) Let $\{A_j : j \in J\}$ be a indexed collection of sets and let B be a set. Which of the following are true: *1 point*

- $B \cup [\bigcup_{j \in J} A_j] = \bigcup_{j \in J}(B \cup A_j)$.
- $B \cup [\bigcap_{j \in J} A_j] = \bigcap_{j \in J}(B \cap A_j)$.
- $B \setminus [\bigcup_{j \in J} A_j] = \bigcap_{j \in J}(B \setminus A_j)$.
- $B \setminus [\bigcap_{j \in J} A_j] = \bigcup_{j \in J}(B \setminus A_j)$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$B \cup [\bigcup_{j \in J} A_j] = \bigcup_{j \in J}(B \cup A_j)$.
$B \setminus [\bigcup_{j \in J} A_j] = \bigcap_{j \in J}(B \setminus A_j)$.
$B \setminus [\bigcap_{j \in J} A_j] = \bigcup_{j \in J}(B \setminus A_j)$.

7) Which of the following is a plausible strategy for proving that a set S is countable (possibly finite). *1 point*

- Show that there is a surjective function from S to the natural numbers.
- Show that some subset of S is countable.
- Find an injective map from S to the natural numbers.
- Decompose S into a countable union of countable sets.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Find an injective map from S to the natural numbers.
Decompose S into a countable union of countable sets.

8) The power set of S often is denoted by 2^S . Which of the following is a plausible explanation for this notation: *1 point*

- Since the power set of a non-empty set always has a larger cardinality, the notation 2^S is natural.
- Any subset of S is uniquely determined by a function $f : S \to \{0, 1\}$ in a natural way and vice-versa.
- If $n \in \mathbb{N}$ then we have

$$2^n = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n}$$

- This notation is easily justified by the law of exponents.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Any subset of S is uniquely determined by a function $f : S \to \{0, 1\}$ in a natural way and vice-versa.
If $n \in \mathbb{N}$ then we have

$$2^n = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n}$$

9) Condier the relation defined on $\mathbb{N} \times \mathbb{N}$ by (m, n) and (r, s) are related if $gcd(m, n) = gcd(r, s)$. Then which of the following is true about the relation: *1 point*

- It is a reflexive relation.
- It is symmetric
- It is transitive
- It is reflexive and symmetric but not transitive.

No, the answer is incorrect.
Score: 0

Accepted Answers:
It is a reflexive relation.
It is symmetric
It is transitive

10) Let $f : A \to B$ be a function and $B_i \subset B (i \in I)$ being some indexing set. Which of the following is true: *1 point*

- $f^{-1}(\bigcup_i B_i) = \bigcup_i f^{-1}(B_i)$.
- $f^{-1}(\bigcap_i B_i) = \bigcap_i f^{-1}(B_i)$.
- $f^{-1}(B \setminus B_1) = A \setminus f^{-1}(B_1)$.
- $f(f^{-1}(B_1)) = B_1$
- if $A_1 \subset A$ then $f^{-1}(f(A_1)) = A_1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f^{-1}(\bigcup_i B_i) = \bigcup_i f^{-1}(B_i)$.
$f^{-1}(\bigcap_i B_i) = \bigcap_i f^{-1}(B_i)$.
$f^{-1}(B \setminus B_1) = A \setminus f^{-1}(B_1)$.