Assignment 9

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due 2020-11-18, 22:59 IST.

Determine the type of isolated singularity at each of the following cases.

1) Consider the function \( f(z) = \frac{1}{z^2-1} \) given by \( f(z) = \frac{1}{z-1} \). Determine the type of singularity at the point \( z = 1 \).
   - Removable singularity
   - Essential singularity
   - No, the answer is incorrect.

2) Consider the function \( f(z) = \frac{1}{z^3+1} \) given by \( f(z) = \frac{1}{z+1} \). Determine the type of singularity at the point \( z = -1 \).
   - Removable singularity
   - Essential singularity
   - No, the answer is incorrect.

3) Consider the function \( f(z) = \frac{1}{z^2+4} \) given by \( f(z) = \frac{1}{z+2i} \). Determine the type of singularity at the point \( z = -2i \).
   - Removable singularity
   - Essential singularity
   - No, the answer is incorrect.

4) Consider the function \( f(z) = \frac{1}{z^2-4} \) given by \( f(z) = \frac{1}{z-2} \). Determine the type of singularity at the point \( z = 2 \).
   - Removable singularity
   - Essential singularity
   - No, the answer is incorrect.

5) Consider the function \( f(z) = \frac{1}{z^2+4} \) given by \( f(z) = \frac{1}{z+2i} \). Determine the type of singularity at the point \( z = -2i \).
   - Removable singularity
   - Essential singularity
   - No, the answer is incorrect.

6) Let \( f(z) \) and \( g(z) \) be two distinct meromorphic functions with poles at \( z = a \) and \( z = b \) respectively at a point \( z \in \mathbb{C} \). Check the branch corresponding to which three statements is true.
   - The point \( z = a \) is an isolated singularity of the function \( \frac{f(z)}{g(z)} \).
   - The function \( \frac{f(z)}{g(z)} \) has a removable singularity at \( z = a \) if \( m > n \).
   - The function \( \frac{f(z)}{g(z)} \) has a pole of order \( m - n \) at \( z = a \) if \( m < n \).
   - The function \( \frac{f(z)}{g(z)} \) has an essential singularity at \( z = a \).
   - No, the answer is incorrect.

7) Check the boxes corresponding to which two statements is true:
   - The Laurent series expansion of the function \( f(z) = \frac{1}{z^3-1} \) has non-zero coefficients with negative indices.
   - The Laurent series expansion of the function \( f(z) = \frac{1}{z^3+1} \) coincides with the power series \( \sum_{n=0}^{\infty} \frac{1}{z^n} \).
   - The Laurent series of the function \( f(z) = \frac{1}{z^3} \) is unique since it has an essential singularity at \( z = 0 \) which is centered at \( 0 \) where the Laurent series expansion is being considered.
   - No, the answer is incorrect.

8) The Laurent series expansion of the function \( f(z) = \frac{1}{z^3-1} \) on \( \mathbb{D}(0) \) coincides with the power series \( \sum_{n=0}^{\infty} \frac{1}{z^{3n}} \).
   - On the annulus \( 1 \leq |z| < 2 \), the function \( f(z) = \frac{1}{z^3-1} \) has a Laurent series expansion \( \sum_{n=0}^{\infty} \frac{1}{z^{3n}} \).
   - No, the answer is incorrect.

9) Consider a function \( f(z) = \frac{1}{z^2+4} \) holomorphic on \( \mathbb{C} \). Determine the type of singularity.
   - The isolated singularity is an essential singularity of \( f(z) = \frac{1}{z^2+4} \).
   - The function \( f(z) = \frac{1}{z^2+4} \) has a removable singularity at \( z = 0 \).
   - The function \( f(z) = \frac{1}{z^2+4} \) has a pole of order 2 at \( z = 0 \).
   - No, the answer is incorrect.