Assignment 8

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-11, 23:59 IST

1) Let $\gamma$ be a closed curve around a point $a_0$ not in the image of $\gamma$. Check the boxes that correspond to true statements. [4 points]

- The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is a locally constant function on $\mathbb{C}$ away from $\gamma$.
- The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is a constant function on $\mathbb{C}$ away from $\gamma$.
- The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is a continuous function on $\mathbb{C}$ away from $\gamma$.
- The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is not real valued.
- The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is an integer.

Note: the answer is incorrect.
Score: 0

Accepted Answer:
The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is a locally constant function on $\mathbb{C}$ away from $\gamma$.
The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is a continuous function on $\mathbb{C}$ away from $\gamma$.
The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is not real valued.
The winding number $W_{\gamma}(a)$ at $a$ around $\gamma$ is an integer.

2) Check the boxes that correspond to functions which are locally invertible. [4 points]

- $f(z) = e^z$ at $z = 0$
- $f(z) = \sin(z^2)$ at $z = 0$
- $f(z) = (\sin(z))^2$ at $z = 0$
- $f(z) = \frac{z - 1}{z - 1/2}$ at $z = 1$.

Note: the answer is incorrect.
Score: 0

Accepted Answers:
$f(z) = e^z$ at $z = 0$
$f(z) = \frac{z - 1}{z - 1/2}$ at $z = 1$.

3) Consider the function $f(z) = z^2$. Check the boxes which correspond to true statements. [4 points]

- The function $f$ is locally invertible at every point in the complex plane.
- The function $f$ is not locally invertible at the origin.
- The function $f$ is locally invertible at every point in $D\setminus\{0\}$.
- The function $f$ is not invertible in $D\setminus\{0\}$.

Note: the answer is incorrect.
Score: 0

Accepted Answer:
The function $f$ is not locally invertible at the origin.
The function $f$ is locally invertible at every point in $D\setminus\{0\}$.
The function $f$ is not invertible in $D\setminus\{0\}$.

4) Let $f: \Omega \to \mathbb{C}$ be a holomorphic function on an open connected subset of $\mathbb{C}$. Check the boxes corresponding to which true statements are given. [4 points]

- If the function $f$ is non-constant, then $f$ is an open mapping.
- If $f'(a_0) \neq 0$, then there exists $\epsilon > 0$ and a neighbourhood $U \subseteq \Omega$ of $a_0$ such that $f$ maps $U$ univocally onto $D(f(a_0), \epsilon)$.
- If $f(\Omega)$ is a closed set in $\mathbb{C}$, then $f$ is a constant function.
- If $f(\Omega)$ is a closed set in $\mathbb{C}$, then $f$ is either a constant function or a surjective function onto the complex plane.

Note: the answer is incorrect.
Score: 0

Accepted Answer:
If the function $f$ is non-constant, then $f$ is an open mapping.
If $f'(a_0) \neq 0$, then there exists $\epsilon > 0$ and a neighbourhood $U \subseteq \Omega$ of $a_0$ such that $f$ maps $U$ univocally onto $D(f(a_0), \epsilon)$.
If $f(\Omega)$ is a closed set in $\mathbb{C}$, then $f$ is either a constant function or a surjective function onto the complex plane.

5) Let $a \in D$ and define $f(z) = \frac{a - z}{1 - a\bar{z}}$. [4 points]

- The function $f$ is holomorphic on an open connected set $\Omega$ which contains the closed unit disk $D$
- If $|a| = 1$, then $|f(z)| = 1$
- If $|a| = 1$, then $|f(z)| \leq 1$
- The image $f(\Omega)$ is an open subset of $\Omega$

Note: the answer is incorrect.
Score: 0

Accepted Answers:
The function $f$ is holomorphic on an open connected set $\Omega$ which contains the closed unit disk $D$
If $|a| = 1$, then $|f(z)| = 1$
If $|a| = 1$, then $|f(z)| \leq 1$
The image $f(\Omega)$ is an open subset of $\Omega$.
The function $f$ maps $D$ to $\mathbb{C}$.