Assignment 12

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

1) Let \( X, Y \subseteq \mathbb{C} \) be open connected and \( p : X \to X \) be a covering map. Check the boxes corresponding to which true statement(s) is/are given.

- For each \( x \in X \) the subset \( p^{-1}(\{x\}) \) of \( Y \) is a discrete subset of \( Y \).
- There exists \( x \in X \) such that the subset \( p^{-1}(\{x\}) \) of \( Y \) has a limit point in \( Y \).
- \( p \) is an open mapping.
- For every open connected subset \( \Omega \subseteq \mathbb{C} \) and a holomorphic function \( g : \Omega \to X \), we can lift the function \( g \) with respect to \( p \) to a holomorphic mapping \( \hat{g} : \Omega \to Y \).

No, the answer is incorrect.

Score: 0
Accepted Answers:
- For each \( x \in X \) the subset \( p^{-1}(\{x\}) \) of \( Y \) is a discrete subset of \( Y \).
- \( p \) is an open mapping.
- Let \( \Omega \subseteq \mathbb{C} \) be an open connected subset which is simply connected and locally connected. Suppose \( g : \Omega \to X \) is a holomorphic function, then we can lift the function \( g \) with respect to \( p \) to a holomorphic mapping \( \hat{g} : \Omega \to Y \).

2) Consider the function \( f : \mathbb{C}^* \to \mathbb{C}^* \) defined by \( f(z) = z^3 \). Let \( \gamma \) be the unit circle given by \( \gamma(t) = e^{it} \) for \( t \in [0, 2\pi] \). Check the boxes corresponding to which true statements are given.

- The function \( f \) is a holomorphic function which is a local homeomorphism with a holomorphic inverse.
- The function \( f \) is a covering map.
- The curve \( \gamma(t) = e^{it} \) for \( t \in [0, 2\pi] \) is a lift of \( \gamma \) with respect to \( f \).
- The curve \( \gamma(t) = e^{3it} \) for \( t \in [0, 2\pi] \) is a lift of \( \gamma \) with respect to \( f \).
- Every closed curve in \( \mathbb{C}^* \) can be lifted to a closed curve with respect to \( f \) in \( \mathbb{C}^* \).

No, the answer is incorrect.

Score: 0
Accepted Answers:
- The function \( f \) is a holomorphic function which is a local homeomorphism with a holomorphic inverse.
- The function \( f \) is a covering map.
- The curve \( \gamma(t) = e^{it} \) for \( t \in [0, 2\pi] \) is a lift of \( \gamma \) with respect to \( f \).

3) Let \( f : D \to \mathbb{C} \) be a holomorphic function on the unit disk such that \( f(0) = 0 \), \( f'(0) = 1 \) and \( |f'(z)| < 2 \) for all \( z \in D \). Check the boxes corresponding to which true statements are given.

- The power series expansion of \( f \) around \( 0 \) be given by \( \sum_{n=0}^{\infty} a_n z^n \). Then \( |a_n| \leq 2 \) for all \( n \geq 2 \).
- On the circle of radius \( 1/4 \), the function \( f \) is bounded below by \( 1/12 \).
- Given \( w \in D(0, 1/12) \), there exists \( z \in D(0, 1/12) \) such that \( f(z) = w \).
- The disk \( D(0, 1/12) \) is contained in the image \( f(D) \).

No, the answer is incorrect.

Score: 0
Accepted Answers:
- The power series expansion of \( f \) around \( 0 \) be given by \( \sum_{n=0}^{\infty} a_n z^n \). Then \( |a_n| \leq 2 \) for all \( n \geq 2 \).
- On the circle of radius \( 1/4 \), the function \( f \) is bounded below by \( 1/12 \).
- Given \( w \in D(0, 1/12) \), there exists \( z \in D(0, 1/12) \) such that \( f(z) = w \).
- The disk \( D(0, 1/12) \) is contained in the image \( f(D) \).

4) Check the boxes corresponding to which true statements are given.

- There exists a non constant entire function \( f = u + iv \) such that for all \( z \in \mathbb{C} \), \( e^{f(z)} \neq 0 \).
- There exists a non constant entire function \( f \) such that \( \Re f \leq 1 \).
- \( f \) which is bounded on the real axis.
- There exists a non constant entire function.
- There exists a non constant entire function \( f \) which is bounded on both the real axis and imaginary axis.
- If \( f \) and \( g \) are entire functions such that \( e^{f(z)} + e^{g(z)} = 1 \) for all \( z \in \mathbb{C} \), then \( f \) and \( g \) are necessarily constant functions.

No, the answer is incorrect.

Score: 0
Accepted Answers:
- There exists a non constant entire function \( f \) which is bounded on the real axis.
- There exists a non constant entire function \( f \) which is bounded on both the real axis and imaginary axis.
- If \( f \) and \( g \) are entire functions such that \( e^{f(z)} + e^{g(z)} = 1 \) for all \( z \in \mathbb{C} \), then \( f \) and \( g \) are necessarily constant functions.