Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

DUE ON 2020-11-25, 23:59 IST.

1) Let \( f(z) = \frac{e^{-z}}{(z-2)^{11}} \) and \( z_0 \) be the residue of \( f(z) \) at the singular point \( z = 2 \). Then the value of \( 6z_0^5 \) is

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) -1

2) Let \( D \) be a domain and \( z_0 \in D \). Suppose \( f(z) \) is a holomorphic functions in \( D \backslash \{z_0\} \) and \( g(z) \) be any holomorphic function in \( D \). Check the boxes corresponding to which true statement(s) are given.

- If \( f(z) \) has a removable singularity at \( z_0 \), then
  \[ \text{Res}_{z_0} f(z) = 0 \]

- If \( f(z) \) has a removable singularity at \( z_0 \) and \( g(z) \neq 0 \), then
  \[ \text{Res}_{z_0} f(z) = \text{Res}_{z_0} g(z) \]

- If \( f(z) \) has a simple pole at \( z_0 \), then
  \[ \text{Res}_{z_0} f(z) = \frac{g(z)}{f(z)} \]

- If \( f(z) \) has a pole of order \( m \geq 2 \) at \( z_0 \) and \( g(z) \neq 0 \), then
  \[ \text{Res}_{z_0} f(z) = \frac{g(z)}{f(z)} \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
- If \( f(z) \) has a removable singularity at \( z_0 \), then
  \[ \text{Res}_{z_0} f(z) = 0 \]

- If \( f(z) \) has a removable singularity at \( z_0 \) and \( g(z) \neq 0 \), then
  \[ \text{Res}_{z_0} f(z) = \text{Res}_{z_0} g(z) \]

- If \( f(z) \) has a simple pole at \( z_0 \), then
  \[ \text{Res}_{z_0} f(z) = \frac{g(z)}{f(z)} \]

- If \( f(z) \) has a pole of order \( m \geq 2 \) at \( z_0 \) and \( g(z) \neq 0 \), then
  \[ \text{Res}_{z_0} f(z) = \frac{g(z)}{f(z)} \]

3) Let \( f(z) \) be a holomorphic function in \( D \backslash \{0\} \). Check the boxes corresponding to which true statement(s) are given.

- Suppose \( f(-z) = f(z) \) for \( z \in D \backslash \{0\} \), then \( \text{Res}_{0} f(z) = 0 \)

- Suppose \( f(-z) = f(z) \) for \( z \in D \backslash \{0\} \), then \( \text{Res}_{z_0} f(z) = -1 \)

- Suppose \( f(-z) = -f(z) \) for \( z \in D \backslash \{0\} \), then \( \text{Res}_{z_0} f(z) = 0 \)

- For each \( a \in \mathbb{C} \), there exists a holomorphic function \( f_a \) on \( D \backslash \{0\} \) such that \( f_a(z) \to f_a(z) \) as \( \text{Res}_{z_0} f_a(z) \to a \)

If \( f(z) \) has pole at \( z = 0 \), then the residue of \( f(z) \) at \( z = 0 \) cannot be zero

No, the answer is incorrect.
Score: 0
Accepted Answers:
- Suppose \( f(-z) = f(z) \) for \( z \in D \backslash \{0\} \), then \( \text{Res}_{0} f(z) = 0 \)

4) Let \( D \) be a domain and \( D \) contains 0. Suppose \( f(z) \) be any holomorphic function on \( D \) such that \( f(z) \neq 0 \) and \( f(z) \neq 0 \) for \( z \in D \). Then the value of the integral \( \frac{1}{2\pi i} \int_{|z|=c} \frac{dz}{z^2} \) is

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 0

5) Let \( f : \Omega \to \mathbb{C} \) be holomorphic functions on an open connected set \( \Omega \) which contains \( D \). Suppose \( f(z) \) has a zero \( z_0 \in \Delta \) and that \( f(z) \) does not vanish at any other point of \( \Delta \). Check the boxes corresponding to which true statement(s) are given.

- The inequality \( |g(z)| < |f(z)| \) for \( z \) on the unit circle is satisfied for \( c > 0 \) small enough.

- For any \( c > 0 \), the function \( f + cg \) does not vanish on the unit circle.

- For \( c > 0 \) small enough, the function \( f + cg \) does not vanish on the unit circle.

- The function \( f + cg \) has a simple zero at \( z_0 \) in \( \Delta \) for any \( c > 0 \).

- For \( c > 0 \) small enough, the function \( f + cg \) has a zero \( z_0 \) in \( \Delta \). Moreover, the zero is simple i.e. \( \text{ord}(f + cg) = 1 \).

No, the answer is incorrect.
Score: 0
Accepted Answers:
The inequality \( |g(z)| < |f(z)| \) for \( z \) on the unit circle is satisfied for \( c > 0 \) small enough.
For \( c > 0 \) small enough, the function \( f + cg \) does not vanish on the unit circle.
For \( c > 0 \) small enough, the function \( f + cg \) has a zero \( z_0 \) in \( \Delta \). Moreover, the zero is simple i.e. \( \text{ord}(f + cg) = 1 \).