Assignment 2

1. Consider the following function:
   \[ f(x) = \begin{cases} 
   2x & \text{if } x \leq 0 \\
   x^2 & \text{if } x > 0 
   \end{cases} 
   \]
   a) Is \( f(x) \) continuous at \( x = 0 \)?
   b) Is \( f(x) \) differentiable at \( x = 0 \)?

2. Find the limit of the sequence \( a_n = \frac{(-1)^n}{n} \) as \( n \to \infty \).

3. Let \( f(x) \) be a continuous function on \( [a, b] \) and differentiable on \( (a, b) \). Prove that there exists at least one point \( c \in (a, b) \) such that:
   \[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

4. Consider the function \( f(x) = \sqrt{x} \) for \( x > 0 \).
   a) Find the derivative of \( f(x) \).
   b) Determine the interval on which \( f(x) \) is increasing.
   c) Find the critical points of \( f(x) \).

5. Let \( f(x) \) be a function with \( f(0) = 0 \) and \( f(1) = 1 \). Define \( g(x) = f(2x) \).
   a) Find \( g(0) \) and \( g(1) \).
   b) Is \( g(x) \) increasing on \( [0, 1] \)?

6. Find the maximum and minimum values of the function \( f(x) = x^3 - 3x + 2 \) on the interval \( [-2, 2] \).

7. Let \( f(x) \) be a twice continuously differentiable function such that \( f(0) = 0 \), \( f'(0) = 1 \), and \( f''(0) = 2 \).
   a) Find \( f(1) \) using the Taylor series expansion up to the second order.
   b) Find the interval in which the error \( |f(x) - f(0) - f'(0)x| \) is less than 0.01 for \( x \) in the interval \( [0, 1] \).