Assignment 6

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-28, 23:59 IST.

1) Let $k$ be an algebraically closed field, $R = k[x_1, \ldots, x_n]$ and $I$ an $R$-ideal. Then the number of irreducible components of $Z(I)$ is at most $\text{deg}(R/I)$. 
   Accepted Answers: True

2) Let $R$ be a noetherian ring and $I$ an $R$-ideal. True/False: If $\text{Ass}(R/I) = \text{Min} R/I$, then $I$ is a radical ideal.
   Accepted Answers: True

3) Let $R = k[x, y, z]$ and $I$ the ideal of the ring map $R \to k[x, y, z]$, $f(x, y, z) \mapsto f(x^2, y^3, z^4)$.
   Pick the correct answer from below:
   - $I : x = (x, y, z)$
   - $I : x^2 = 0$
   - $I : x^3 = 0$
   Accepted Answers: True

4) Let $(R, m)$ be a noetherian local ring and $I$ an $R$-ideal such that $\sqrt{I} \neq m$. True/False: $m$ contains a non-zero-divisor on $R/(I : m^n)$.
   Accepted Answers: True

5) True/False: In the ring $k[x, y, z, w]$, $(x, y) \cap (x, w) = (x, y, z, xw, yw)$.
   Accepted Answers: True

6) True/False: If $I : w^n = I : w^{n+1}$, then $I : w^n = I : w^n$.
   Accepted Answers: True

7) Consider the ring map $C[y, z] \to C[x, y]$, $f(x, y) \mapsto f(x^2, y^3)$. The fibre over the point corresponding to $(x - \alpha, y - \beta)$ is given by the maximal ideal $\mathfrak{p} = (\alpha - x, \beta - y)$. Is $\mathfrak{p}$ given by the maximal ideal $(\alpha - x, \beta - y)$ if $\alpha \neq 0$?
   Accepted Answers: True

8) Let $\varphi : R \to S$ be a map of rings, $y \in \text{Spec} R$ and $q \in \text{Spec} S$ such that $\varphi^{-1}(q) = y$. True/False: Then the fraction field of $S/y$ is a finite extension of the fraction field of $R/y$.
   Accepted Answers: False

9) True/False: Let $R$ be a domain and $a, b \in R$, $b \neq 0$. Then $R[\frac{a}{b}]$ is never integral over $R$.
   Accepted Answers: True

10) True/False: For every positive integer $n$, $\sqrt[n]{x}$ is integral over $\mathbb{Z}$.
    Accepted Answers: True