Assignment 2

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

1) Let $k$ be a field and $R = k[X_1, X_2, X_3]$. Give $R$ the lexicographic monomial order. Let $f = X_1^2 + X_1X_2^3 + X_1^3 + X_2X_3^2$. Then the initial term of $f$ is $X_1^2$. (1 point)

2) In $k[X_1, X_2, X_3]$, with lexicographic order, the set $\{X_1 - X_2^2, X_2 - X_1^3\}$ is a Grobner basis for the ideal generated by it. (1 point)

3) Let $R = k[X_1, X_2, \ldots, X_n]$, $m = (X_1, \ldots, X_n)$, $f_1, \ldots, f_n \in m^2$. Let $G$ be a Grobner basis of $f$ with respect to the degree-lexicographic order. True/False: For each $g \in G$, the constant term of $g$ is zero. (1 point)

4) The cardinality of a minimal generating set of the ideal $(x^2, x^2y, x^2z, x^2z^2)$ of the ring $k[x, y, z]$ is 4. (1 point)

5) In $C[x, y, z]$, the ideal $(x + 1, y + 1, z)$ contains the ideal $(x, y, z^2 + y, xy + x, xy - y)$. (1 point)

6) A Grobner basis for the ideal $(x + 1, y^2 + x + xy, x^2 - y + 1)$ in the degree-reverse-lexicographic order contains 1. (1 point)

7) $(z + 1, x^2 + y, xy + x, xy - y)$ is a maximal ideal of $Q[x, y, z]$. (1 point)

8) What is the number of maximal ideals $m$ of $R = Q[x]/(x^2 + 1)$ for which $R/m \cong Q$? (1 point)

9) What is the number of points in the variety of the ideal $(x^2 + 1)$ in the ring $C[x]$? (1 point)

10) What is the number of points in the variety of the ideal $(x^2 + 1)$ in the ring $k[x]$, where $k$ is a field of characteristic 2? (1 point)