Unit 10 - Week 8

Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

Week 3

Week 4

Week 5

Week 6

Week 7

Week 8

- Simulation of stochastic differential equations
- Stochastic differential equations: Uniqueness
- Stochastic differential equations: Existence part 1
- Stochastic differential equations: Existence part 2
- Stochastic differential equations: Existence part 3
- Stochastic differential equations: Weak solution
- Quiz : Assignment 8
- Probabilistic Methods in PDE: Week 8 Feedback form

Week 9

Week 10

Week 11

Week 12

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Assignment 8

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-03-25, 23:59 IST.

1) Identify the correct statements. Let $u \in C[a,b] \cap C^1(a,b)$ where $0 < a < b$. If $u(t) \leq -\int_a^t u(t)dt$ for all $t \in [a,b]$ then $u(t)$ is less than or equal to *2 points*

- $u(a)e^{a-t}$
- $u(a)e^{t-a}$
- $u(a)(t-a)$
- $\frac{u(a)}{e^a}e^{-t}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(a)e^{a-t}$
$u(a)e^{t-a}$

2) Let $f : (0,\infty) \to \mathbb{R}$. Which of the following functions have at most linear growth? *2 points*

- $f(x) = x\sin(x)$
- $f(x) = e^x$
- $f(x) = \frac{1+x^2}{1+x}$
- $f(x) = \frac{x^2e^{-x}}{1+x}$
- $f(x) = x\ln x$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f(x) = x\sin(x)$
$f(x) = \frac{1+x^2}{1+x}$
$f(x) = \frac{x^2e^{-x}}{1+x}$

3) Let $f : (0,\infty) \to \mathbb{R}$. Which of the following functions have at most quadratic growth? *2 points*

- $f(x) = x\ln x$
- $f(x) = x(\ln x)^3$
- $f(x) = \frac{1+x^2}{x}$
- $f(x) = \frac{1+x^2}{e^{-x}}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f(x) = x\ln x$
$f(x) = x(\ln x)^3$

4) let $p, q \in [1,\infty]$ with $1/p + 1/q = 1$. If $f \in L^p[0,1]$ and $g \in L^q[0,1]$, then identify the correct statements *1 point*

- $\int_0^1 |fg|(x)dx < \infty$
- $\int_0^1 |fg|^{\frac{p+q}{pq}}(x)dx < \infty$
- $\int_0^1 |fg|(x)dx \leq (\int_0^1 |f|^p(x)dx)^{1/p}(\int_0^1 |g|^q(x)dx)^{1/q}$
- $\int_0^1 |f|(x)dx \leq (\int_0^1 |f|^p(x)dx)^{1/p}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_0^1 |fg|(x)dx < \infty$
$\int_0^1 |fg|^{\frac{p+q}{pq}}(x)dx < \infty$
$\int_0^1 |fg|(x)dx \leq (\int_0^1 |f|^p(x)dx)^{1/p}(\int_0^1 |g|^q(x)dx)^{1/q}$
$\int_0^1 |f|(x)dx \leq (\int_0^1 |f|^p(x)dx)^{1/p}$

5) Let $sign(x)$ give $+1, 0$ or -1 if x is positive, zero or negative respectively. Define an odd function $f(x) := sign(x)\sum_{n=1}^{\infty} 1_{[-1/n,1/n]}(x)$. Let $F(x) := \int_{-\infty}^{-|x|} f(y)dy$ for $x \neq 0$ and $F(0) = 0$. Find the correct statements. *1 point*

- $f \in L^1(\mathbb{R})$
- f is locally integrable
- f is discontinuous at zero
- F has removable discontinuity at 0

No, the answer is incorrect.
Score: 0

Accepted Answers:
f is discontinuous at zero

6) Given a filtered probability space and a Brownian motion W, consider the stochastic differential equation *1 point*

$$dX_t = \mu X_t dt + \sigma dW_t, X_0 = 0$$

where μ and σ are constants. This equation is often referred to as Langenvin equation. Identify the correct statements.

- The SDE has a nonzero strong solution
- All the coefficients of the SDE satisfy the Lipschitz continuity and the at most linear growth property
- The SDE has a weak solution
- The solution is positive for all $t > 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
The SDE has a nonzero strong solution
All the coefficients of the SDE satisfy the Lipschitz continuity and the at most linear growth property
The SDE has a weak solution

7) Given a filtered probability space and a Brownian motion W, consider the stochastic differential equation *1 point*

$$dX_t = \mu X_t dt + \sigma X_t dW_t, X_0 = 0$$

where μ and σ are constants. his equation is called the geometric Brownian motion equation. Identify the correct statements

- The SDE has a nonzero strong solution
- All the coefficients of the SDE satisfy the Lipschitz continuity and the at most linear growth property
- The SDE has a weak solution
- The solution is unique

No, the answer is incorrect.
Score: 0

Accepted Answers:
All the coefficients of the SDE satisfy the Lipschitz continuity and the at most linear growth property
The SDE has a weak solution
The solution is unique