Unit 8 - Week 6

Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

Week 3

Week 4

Week 5

Week 6

- Uniqueness of solution to the heat equation
- Remark on Tychonoff's Theorem
- Widder's result and its extension on heat equation
- Solution to the mixed initial boundary value problem
- The Feynman-Kac formula
- Quiz : Assignment 6
- Probabilistic Methods in PDE: Week 6 Feedback form

Week 7

Week 8

Week 9

Week 10

Week 11

Week 12

DOWNLOAD VIDEOS

Assignment 6

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-03-11, 23:59 IST.

Let τ_D be the exit time from the set D by the Brownian motion W

1) Define $f(x) := E[1_{\{0\}}(W_{\tau_{(0,\infty)}}) \mid W_0 = x]$ for all $x \in \mathbb{R}$. Identify the correct statements *3 points*

- $f(x) = 1_{\{0\}}(x)$ for all x
- $f(x) = 0$ for all x
- $f(x) = 1_{[0,\infty)}(x)$ for all x
- $\lim_{x\uparrow 0} f(x) = 0$
- $\lim_{x\downarrow 0} f(x) = 0$
- $f : \mathbb{R} \to \mathbb{R}$ is continuous
- $f : \mathbb{R} \to \mathbb{R}$ is bounded
- $f(x) \leq 1$ for all x
- f is continuous at the boundary of $(0, \infty)$
- The boundary of $(0, \infty)$ is regular

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f(x) = 1_{[0,\infty)}(x)$ *for all* x
$\lim_{x\uparrow 0} f(x) = 0$
$f : \mathbb{R} \to \mathbb{R}$ *is bounded*
$f(x) \leq 1$ *for all* x
The boundary of $(0, \infty)$ *is regular*

2) If D is an open connected domain and $a \in \partial D$ regular for D, then find out the correct statements *2 points*

- $\lim_{n\to\infty} \lim_{\substack{x\to a \\ x\in D}} P(\tau_D > 1/n | W_0 = x) = 0$
- $\lim_{\substack{x\to a \\ x\in D}} \lim_{n\to\infty} P(\tau_D > 1/n | W_0 = x) = 0$
- $\lim_{\substack{x\to a \\ x\in D}} E(f(W_{\tau_D}) | W_0 = x) = f(a)$ for all f bounded continuous functions
- $\lim_{\substack{x\to a \\ x\in D}} P[\tau_D = \infty | W_0 = x] = 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\lim_{n\to\infty} \lim_{\substack{x\to a \\ x\in D}} P(\tau_D > 1/n | W_0 = x) = 0$
$\lim_{\substack{x\to a \\ x\in D}} E(f(W_{\tau_D}) | W_0 = x) = f(a)$ *for all* f *bounded continuous functions*
$\lim_{\substack{x\to a \\ x\in D}} P[\tau_D = \infty | W_0 = x] = 0$

3) Which of the following functions $f : \mathbb{R} \to \mathbb{R}$ satisfy the condition *3 points*

$$\int_{-\infty}^{\infty} |f(x)| e^{-ax^2}\, dx < \infty \text{ for some } a > 0$$

- $f(x) = x^{100}$
- $f(x) = 100^x$
- $f(x) = x^x$
- $f(x) = (1/e)^{x^3}$
- $f(x) = e^{x^3}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f(x) = x^{100}$
$f(x) = 100^x$
$f(x) = (1/e)^{x^3}$

4) Let f be a bounded continuous function and u solves *2 points*

$$\frac{\partial u}{\partial t}(t, x) = \frac{1}{2} \frac{\partial^2 u}{\partial x^2}(t, x) \text{ with } u(0, x) = f(x), \ x \in \mathbb{R}.$$

If W is the standard Brownian motion, identify the correct statements, if any.

- $u(t, x) = E[f(x + W_t) | W_0 = 0]$
- $u(t, x) = E[|f(x + W_t)| | W_0 = 0]$
- $u(t, x) = E[f(W_t) | W_0 = x]$
- $u(t, x) = E[f(W_t) | W_t = x]$
- $u(t, x) = E[f(x + W_{t+s}) | W_s = 0]$ for any $s > 0$
- $u(t, x) = E[f(-W_t) | W_0 = -x]$
- $u(t, x) = E[f(x - W_t) | W_0 = 0]$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$u(t, x) = E[f(x + W_t) | W_0 = 0]$
$u(t, x) = E[f(W_t) | W_0 = x]$
$u(t, x) = E[f(x + W_{t+s}) | W_s = 0]$ *for any* $s > 0$
$u(t, x) = E[f(-W_t) | W_0 = -x]$
$u(t, x) = E[f(x - W_t) | W_0 = 0]$