Unit 7 - Week 5

Assignment 5

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-03-04, 23:59 IST.

1) Let D be an open subset of $\mathbb{R}^d$. If $u : D \to \mathbb{R}$ is such that $u \in C^2(D)$ and $\Delta u = 0$ in D. Then *4 points*

- u is harmonic in B for every open subset B of D
- u has Mean Value Property in D
- for every $a \in D$ and $r > 0$ s.t. $a + \bar{B}_r \subset D$, one has $u(a) = \int_{\partial B_r} u(a+x)\mu_r(dx)$
- $u(a) = \frac{1}{|B_r|}\int_{B_r} u(a+x)\,dx$, where | | is the d-dimensional Lebesgue measure
- u is harmonic in the closure of D
- there is a point $y \in D$ such that $u(y) = \sup_{x\in D} u(x)$
- $u \in C^\infty(D)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
u is harmonic in B for every open subset B of D
u has Mean Value Property in D
for every $a \in D$ and $r > 0$ s.t. $a + \bar{B}_r \subset D$, one has $u(a) = \int_{\partial B_r} u(a+x)\mu_r(dx)$
$u(a) = \frac{1}{|B_r|}\int_{B_r} u(a+x)\,dx$, where | | is the d-dimensional Lebesgue measure
$u \in C^\infty(D)$

2) Let D be an open subset of d-dimensional space ($D \Subset \mathbb{R}^d$) and $f : \partial D \to \mathbb{R}$ continuous.Let τ_D be the exit time from the set D by the Brownian motion W.If $E[|f(W_{\tau_D})| \mid W_0 = x] < \infty \; \forall\, x \in D$, then $u(x) = E[f(W_{\tau_D})|W_0 = x]$ *3 points*

- is harmonic in D
- has Mean Value Property in D
- is in $C^\infty(D)$
- solves the Dirichlet problem (D, f)
- is in $C(\bar{D})$

No, the answer is incorrect.
Score: 0

Accepted Answers:
is harmonic in D
has Mean Value Property in D
is in $C^\infty(D)$

3) Let τ_{D_n} be the exit time from the domain $D_n := \left\{ x \in D \mid \inf_{y\in\partial D} \|x-y\| > 1/n \right\}$. As $n \to \infty$, τ_{D_n} *3 points*

- diverges to ∞
- converges to τ_D a.s.
- converges to 0 a.s.
- none of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
converges to τ_D a.s.