

# Unit 7 - Week 5

## Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

Week 3

Week 4

Week 5

- Summary of the Zaremba's cone condition
- Continuity of candidate solution at regular points - Part 1
- Continuity of candidate solution at regular points - Part 2
- Summary of bounded solution to the Dirichlet Problem
- Stochastic representation of bounded solution to a heat equation - Part 1
- Stochastic representation of bounded solution to a heat equation - Part 2
- Quiz : Assignment 5**
- Probabilistic Methods in PDE: Week 5 Feedback form

Week 6

Week 7

Week 8

Week 9

Week 10

Week 11

Week 12

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## Assignment 5

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

**Due on 2020-03-04, 23:59 IST.**

1) Let  $D$  be an open subset of  $\mathbb{R}^d$ . If  $u : D \rightarrow \mathbb{R}$  is such that  $u \in C^2(D)$  and  $\Delta u = 0$  in  $D$ . Then

4 points

- $u$  is harmonic in  $B$  for every open subset  $B$  of  $D$
- $u$  has Mean Value Property in  $D$
- for every  $a \in D$  and  $r > 0$  s.t.  $a + \bar{B}_r \subset D$ , one has  $u(a) = \int_{\partial B_r} u(a+x) \mu_r(dx)$
- $u(a) = \frac{1}{|B_r|} \int_{B_r} u(a+x) dx$ , where  $| \cdot |$  is the  $d$ -dimensional Lebesgue measure
- $u$  is harmonic in the closure of  $D$
- there is a point  $y \in D$  such that  $u(y) = \sup_{x \in D} u(x)$
- $u \in C^\infty(D)$

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**  
 $u$  is harmonic in  $B$  for every open subset  $B$  of  $D$   
 $u$  has Mean Value Property in  $D$   
 for every  $a \in D$  and  $r > 0$  s.t.  $a + \bar{B}_r \subset D$ , one has  $u(a) = \int_{\partial B_r} u(a+x) \mu_r(dx)$   
 $u(a) = \frac{1}{|B_r|} \int_{B_r} u(a+x) dx$ , where  $| \cdot |$  is the  $d$ -dimensional Lebesgue measure  
 $u \in C^\infty(D)$

2) Let  $D$  be an open subset of  $d$ -dimensional space ( $D \Subset \mathbb{R}^d$ ) and  $f : \partial D \rightarrow \mathbb{R}$  continuous. Let  $\tau_D$  be the exit time from the set  $D$  by the Brownian motion  $W$ . If  $E[|f(W_{\tau_D})| | W_0 = x] < \infty \forall x \in D$ , then  $u(x) = E[f(W_{\tau_D}) | W_0 = x]$

3 points

- is harmonic in  $D$
- has Mean Value Property in  $D$
- is in  $C^\infty(D)$
- solves the Dirichlet problem  $(D, f)$
- is in  $C(\bar{D})$

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**  
 $u$  is harmonic in  $D$   
 $u$  has Mean Value Property in  $D$   
 $u$  is in  $C^\infty(D)$

3) Let  $\tau_{D_n}$  be the exit time from the domain  $D_n := \left\{ x \in D \mid \inf_{y \in \partial D} \|x - y\| > 1/n \right\}$ . As  $n \rightarrow \infty$ ,  $\tau_{D_n}$

3 points

- diverges to  $\infty$
- converges to  $\tau_D$  a.s.
- converges to 0 a.s.
- none of the above

No, the answer is incorrect.  
Score: 0

**Accepted Answers:**  
 converges to  $\tau_D$  a.s.