

Unit 2 - Week 0

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Assignment 0

The due date for submitting this assignment has passed. **Due on 2020-02-05, 23:59 IST.**
 As per our records you have not submitted this assignment.

Identify the correct statements from the following list, if any.

1) Let X be a nonempty set and \mathcal{C} a collection of subsets of X . Then there is a smallest σ algebra containing \mathcal{C} 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

2) There is a countably infinite σ algebra 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

3) There is a countable subset of \mathbb{R} which is not Lebesgue measurable 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

4) There is a pair of non-measurable sets A and B such that $A \cup B$ is measurable 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

5) If A be a non Lebesgue measurable set, $m^*(A) > 0$, where m^* is outer-measure 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

6) There is a collection of finitely many disjoint sets A_1, \dots, A_n such that $m^*(\bigcup_{i=1}^n A_i) < \sum_{i=1}^n m^*(A_i)$, where m^* is outer-measure 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

7) Given $\epsilon > 0$, there is an open set U , containing all of rational numbers such that $m(U) < \epsilon$ 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

Let E be a Lebesgue measurable subset of \mathbb{R} and $f : \mathbb{R} \rightarrow \mathbb{R}$ a continuous function. Then

8) $f(E)$ is Lebesgue measurable 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

9) $f^{-1}(E)$ is Lebesgue measurable 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

10) Let function $f : \mathbb{R} \rightarrow [0, 1]$ be defined by $f(x) = 1_{(0, \infty)}(x)$. If the domain \mathbb{R} and the codomain $[0, 1]$ be endowed with the σ -algebra $\{\emptyset, \mathbb{R}\}$ and Borel σ -algebra respectively then f is measurable 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

11) Let $f : [0, 1] \rightarrow \mathbb{R}$ defined by $f(x) = x^2$. If the domain is endowed with $\{\emptyset, [0, \frac{1}{2}], [\frac{1}{2}, 1], [0, 1]\}$ and the codomain with the Borel σ -algebra, then f is non-measurable function 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

12) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and $g : \mathbb{R} \rightarrow \mathbb{R}$ Lebesgue measurable then $g \circ f$ is Lebesgue measurable 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

13) Let A be a countable union of connected subsets of \mathbb{R} , then $\mathbb{R} \setminus A$ is also a countable union of connected sets. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

Let $\{f_n\}$ be a sequence of real valued Lebesgue integrable functions defined on \mathbb{R} . Also let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a Lebesgue integrable function.

Identify the correct statements from the following list, if any.

14) If $0 \leq f_n$ for all n and $f_n \rightarrow f$ a.e. then $\liminf \int f_n = \int f$. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

15) If $0 \leq f_n \leq f$ for all n and $f_n \rightarrow f$ a.e. then $\liminf \int f_n = \int f$. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

16) If $0 \leq f_n$ for all n and $f_n \rightarrow 0$ a.e. then $\liminf \int f_n = 0$. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

17) If $0 \leq f_n$ for all n and $f_n \downarrow f$ a.e. then $\liminf \int f_n = \int f$. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

18) If $0 \leq f_n$ is Riemann Integrable for all n and $f_n \uparrow f$ a.e. then f is also Riemann Integrable. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

19) If $\lim_{n \rightarrow \infty} m\{x \in \mathbb{R} : |f_n(x) - f(x)| > \epsilon\} = 0$ for all $\epsilon > 0$ then $f_n \rightarrow f$ a.e. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

20) $f_n \rightarrow f$ a.e. and $\int f_n \rightarrow \int f$, then $\int |f - f_n| \rightarrow 0$. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False

Let $f_n = n 1_{(0, \frac{1}{n}]}$, $g_n = \frac{1}{n} 1_{[0, n]}$ and $h_n = 1_{[n-1, n]}$.

Find

21) $\lim \int f_n$ 1 point

0
 1

No, the answer is incorrect.
 Score: 0
 Accepted Answers: 1

22) $\int \lim f_n$ 1 point

0
 1

No, the answer is incorrect.
 Score: 0
 Accepted Answers: 0

23) $\lim \int g_n$ 1 point

0
 1

No, the answer is incorrect.
 Score: 0
 Accepted Answers: 1

24) $\int \lim g_n$ 1 point

0
 1

No, the answer is incorrect.
 Score: 0
 Accepted Answers: 0

25) $\lim \int h_n$ 1 point

0
 1

No, the answer is incorrect.
 Score: 0
 Accepted Answers: 1

26) $\int \lim h_n$ 1 point

0
 1

No, the answer is incorrect.
 Score: 0
 Accepted Answers: 0

Let $f : [0, 1] \rightarrow \mathbb{R}$, $A \subset \mathbb{R}$ and $B := f^{-1}(A)$. Identify the correct statements from the following list.

27) If f is continuous function and A is open then B is open 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

28) If f is Lebesgue measurable function and A is Borel measurable then B is Lebesgue measurable. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

29) If f is a step function then for any A , B can be written as union of a disjoint collection of finitely many connected sets. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: True

30) If f is a simple function then for any A , B can be written as union of a disjoint collection of finitely many connected sets. 1 point

True
 False

No, the answer is incorrect.
 Score: 0
 Accepted Answers: False