

Unit 4 - Week 2

Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

- Preliminary for Stochastic Integration - Part 01
- Preliminary for Stochastic Integration - Part 02
- Definition and properties of Stochastic Integration - Part 01
- Definition and properties of Stochastic Integration - Part 02
- Further properties of Stochastic Integration

Quiz : Assignment 2

- Probabilistic Methods in PDE: Week 2 Feedback form

Week 3

Week 4

Week 5

Week 6

Week 7

Week 8

Week 9

Week 10

Week 11

Week 12

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Assignment 2

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-02-19, 23:59 IST.

1) Consider the probability space $([0, 1], \mathcal{B}, m)$, where m is the Lebesgue measure. Identify the options where the random variable Y is in $L^2([0, 1])$. **1 point**

$Y(\omega) = \sum_{n=1}^{\infty} X_n(\omega)$, where $X_n = 2^n \cdot \mathbb{1}_{(\frac{1}{2^n}, \frac{1}{2^{n-1}}]}(\omega)$

$Y(\omega) = \sum_{n=1}^{\infty} X_n(\omega)$, where $X_n = \frac{2^n}{n} \mathbb{1}_{(\frac{1}{2^n}, \frac{1}{2^{n-1}}]}(\omega)$

$Y(\omega) = \sum_{n=1}^{\infty} X_n(\omega)$, where $X_n = \frac{2^{\frac{n}{2}}}{n} \mathbb{1}_{(\frac{1}{2^n}, \frac{1}{2^{n-1}}]}(\omega)$

none of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:

$Y(\omega) = \sum_{n=1}^{\infty} X_n(\omega)$, where $X_n = \frac{2^{\frac{n}{2}}}{n} \mathbb{1}_{(\frac{1}{2^n}, \frac{1}{2^{n-1}}]}(\omega)$

2) Which of the following random variables are stopping time w.r.t. the filtration $\{\mathcal{F}_n\}_n$, where $\mathcal{F}_n = \sigma(X_1, X_2, \dots, X_n)$ and X_i are independent *Bernoulli*(1/2) random variables **1 point**

$T(\omega) = 5 \forall \omega$

$T(\omega) = \min\{n \geq 1 | X_n(\omega) = 1\}$

$T(\omega) = n$, where $\sum_{i=1}^m X_i(\omega) < 100 \leq \sum_{i=1}^n X_i(\omega)$ for any $m < n$

$T(\omega) = \sqrt{\min\{n \geq 1 | X_n(\omega) = 1\}}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$T(\omega) = 5 \forall \omega$

$T(\omega) = \min\{n \geq 1 | X_n(\omega) = 1\}$

$T(\omega) = n$, where $\sum_{i=1}^m X_i(\omega) < 100 \leq \sum_{i=1}^n X_i(\omega)$ for any $m < n$

3) Let $S = \{S_n\}_n$ be given below. Which of the following are martingales? **1 point**

$S_n = \sum_{i=1}^n X_i$, where for $i \geq 1$, $X_i \stackrel{iid}{=} \begin{cases} 1 & \text{with probability } \frac{1}{2} \\ -1 & \text{with probability } \frac{1}{2} \end{cases}$

$S_n = \prod_{i=1}^n X_i$, where for $i \geq 1$, $X_i \stackrel{iid}{=} \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 2 & \text{with probability } \frac{1}{2} \end{cases}$

$S_n = \sum_{i=1}^n X_i$, where for $i \geq 1$, X_i is *Bernoulli*(1/2)

$S_n = n$ with probability 1, for every $n \geq 1$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$S_n = \sum_{i=1}^n X_i$, where for $i \geq 1$, $X_i \stackrel{iid}{=} \begin{cases} 1 & \text{with probability } \frac{1}{2} \\ -1 & \text{with probability } \frac{1}{2} \end{cases}$

$S_n = \prod_{i=1}^n X_i$, where for $i \geq 1$, $X_i \stackrel{iid}{=} \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 2 & \text{with probability } \frac{1}{2} \end{cases}$

4) Consider a square integrable continuous martingale $M := \{M_t\}_{t \geq 0}$. Define $X = (M^2 - \langle M \rangle)^2$. Identify the correct option(s) **1 point**

X_t is a martingale

X_t is a sub-martingale

X_t is a super-martingale

None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:

X_t is a sub-martingale

5) Let $M(\neq 0) \in \mathcal{M}_t^c$. If $X \in \mathcal{L}^*(M)$ such that $\langle I(X) \rangle \neq 0$, then which of the following is true? **1 point**

X_t is independent of M_t for each $t \geq 0$

X is not identically zero

$I(X)$ is not a martingale

$I(X)$ is not a square integrable continuous martingale

No, the answer is incorrect.
Score: 0

Accepted Answers:

X is not identically zero