Unit 4 - Week 2

Assignment 2

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-02-19, 23:59 IST.

1) Consider the probability space $([0,1], \mathcal{B}, m)$, where m is the Lebesgue measure. Identify the options where the random variable Y is in $L^2([0,1])$. *1 point*

- $Y(\omega) = \sum_{n=1}^{\infty} X_n(\omega)$, where $X_n = 2^n . \mathbb{1}_{(\frac{1}{2^n}, \frac{1}{2^{n-1}}]}(\omega)$
- $Y(\omega) = \sum_{n=1}^{\infty} X_n(\omega)$, where $X_n = \frac{2^n}{n} \mathbb{1}_{(\frac{1}{2^n}, \frac{1}{2^{n-1}}]}(\omega)$
- $Y(\omega) = \sum_{n=1}^{\infty} X_n(\omega)$, where $X_n = \frac{2^{\frac{n}{2}}}{n} \mathbb{1}_{(\frac{1}{2^n}, \frac{1}{2^{n-1}}]}(\omega)$
- none of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
$Y(\omega) = \sum_{n=1}^{\infty} X_n(\omega)$, where $X_n = \frac{2^{\frac{n}{2}}}{n} \mathbb{1}_{(\frac{1}{2^n}, \frac{1}{2^{n-1}}]}(\omega)$

2) Which of the following random variables are stopping time w.r.t. the filtration $\{\mathcal{F}_n\}_n$, where $\mathcal{F}_n = \sigma(X_1, X_2, \ldots, X_n)$ and X_i are independent $Bernouli(1/2)$ random variables *1 point*

- $T(\omega) = 5 \; \forall \; w$
- $T(\omega) = \min\{n \geq 1 | X_n(\omega) = 1\}$
- $T(\omega) = n$, where $\sum_{i=1}^{m} X_i(\omega) < 100 \leq \sum_{i=1}^{n} X_i(\omega)$ for any $m < n$
- $T(\omega) = \sqrt{\min\{n \geq 1 | X_n(\omega) = 1\}}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$T(\omega) = 5 \; \forall \; w$
$T(\omega) = \min\{n \geq 1 | X_n(\omega) = 1\}$
$T(\omega) = n$, where $\sum_{i=1}^{m} X_i(\omega) < 100 \leq \sum_{i=1}^{n} X_i(\omega)$ for any $m < n$

3) Let $S = \{S_n\}_n$ be given below. Which of the following are martingales? *1 point*

- $S_n = \sum_{i=1}^{n} X_i$, where for $i \geq 1$, $X_i \stackrel{iid}{=} \begin{cases} 1 & \text{with probability } \frac{1}{2} \\ -1 & \text{with probability } \frac{1}{2} \end{cases}$
- $S_n = \prod_{i=1}^{n} X_i$, where for $i \geq 1$, $X_i \stackrel{iid}{=} \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 2 & \text{with probability } \frac{1}{2} \end{cases}$
- $S_n = \sum_{i=1}^{n} X_i$, where for $i \geq 1$, X_i is $Bernoulli(1/2)$
- $S_n = n$ with probability 1, for every $n \geq 1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$S_n = \sum_{i=1}^{n} X_i$, where for $i \geq 1$, $X_i \stackrel{iid}{=} \begin{cases} 1 & \text{with probability } \frac{1}{2} \\ -1 & \text{with probability } \frac{1}{2} \end{cases}$
$S_n = \prod_{i=1}^{n} X_i$, where for $i \geq 1$, $X_i \stackrel{iid}{=} \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 2 & \text{with probability } \frac{1}{2} \end{cases}$

4) Consider a square integrable continuous martingale $M := \{M_t\}_{t \geq 0}$. Define $X = (M^2 - \langle M \rangle)^2$. Identify the correct option(s) *1 point*

- X_t is a martingale
- X_t is a sub-martingale
- X_t is a super-martingale
- None of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
X_t is a sub-martingale

5) Let $M(\neq 0) \in \mathcal{M}_2^c$. If $X \in \mathcal{L}^*(M)$ such that $\langle I(X) \rangle \neq 0$, then which of the following is true? *1 point*

- X_t is independent of M_t for each $t \geq 0$
- X is not identically zero
- $I(X)$ is not a martingale
- $I(X)$ is not a square integrable continuous martingale

No, the answer is incorrect.
Score: 0
Accepted Answers:
X is not identically zero