

Unit 14 - Week 12

Course outline

How does an NPTEL online course work?

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Sufficient condition for existence of an evolution system

Y-valued solution

mild/generalized solution to Semi-linear Evolution Problem

Existence of classical solution part 1

Existence of classical solution part 2

Conclusion video

Quiz : Assignment 12

Probabilistic Methods in PDE: Week 12 Feedback form

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Assignment 12

The due date for submitting this assignment has passed. **Due on 2020-04-22, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Identify the necessary conditions for existence of the mild solution on $[s, T]$ of the following inhomogeneous evolution problem **2 points**

$$\frac{d\varphi}{dt} = A(t)\varphi(t) + f(t) \quad \forall t \in (s, T], \varphi(s) = x$$

- integrability of f
- continuous differentiability of f
- $D(A(t)) = X$ and $\|A(t)\|_{BL(X)} < \infty$ for all $t \in [0, T]$
- existence of an ES associated with $\{A(t)\}_{t \in [0, T]}$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
integrability of f
existence of an ES associated with $\{A(t)\}_{t \in [0, T]}$.

2) Assume that A is the infinitesimal generator of a C_0 semigroup $\{T(t)\}_{t \geq 0}$ on X . If $\varphi : [s, T] \rightarrow X$ is the mild solution to $\frac{d\varphi(t)}{dt} = A\varphi(t) + f(t, \varphi(t))$ for $t \in (s, T]$ $\varphi(s) = x$, (sEP) **2 points**

Identify the correct statement(s), if any.

- φ solves $\varphi(t) = T(t-s)x + \int_s^t T(t-r)f(r, \varphi(r))dr$
- $\varphi(t) = T(t-s)x + \int_s^t T(t-r)f(r, x)dr$
- $\varphi(t) = T(t-s)x$
- $\varphi(t) = T(t)x$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 φ solves $\varphi(t) = T(t-s)x + \int_s^t T(t-r)f(r, \varphi(r))dr$

3) Assume that A is the infinitesimal generator of a C_0 semigroup $\{T(t)\}_{t \geq 0}$ on X . Let φ be the continuous solution to $\varphi(t) = T(t-s)x + \int_s^t T(t-r)f(r, \varphi(r))dr$, **2 points**

then $\frac{d\varphi}{dt}$, if exists, is equal to

- $A\varphi(t) + f(t, \varphi(t))$
- $\frac{\partial}{\partial t} f(t, \varphi(t)) + \frac{\partial}{\partial v} f(s, v)|_{v=\varphi(t)}$

$$w(t) = g(t) + \int_s^t T(t-r) \frac{\partial}{\partial v} f(s, v)|_{v=\varphi(r)} w(r) dr$$

w where w solves

$$g(t) = T(t-s)f(s, \varphi(s)) + AT(t-s)x + \int_s^t T(t-r) \frac{\partial}{\partial t} f(r, \varphi(r)) dr.$$

None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $A\varphi(t) + f(t, \varphi(t))$

$$w(t) = g(t) + \int_s^t T(t-r) \frac{\partial}{\partial v} f(s, v)|_{v=\varphi(r)} w(r) dr$$

w where w solves

$$g(t) = T(t-s)f(s, \varphi(s)) + AT(t-s)x + \int_s^t T(t-r) \frac{\partial}{\partial t} f(r, \varphi(r)) dr.$$

4) Let $[s, T] \times X \ni (t, x) \mapsto f(t, x) \in X$ be continuous in t and uniformly Lipschitz in x . Also assume that A is the infinitesimal generator of a C_0 semigroup $\{T(t)\}_{t \geq 0}$ on X . Let $\varphi^{(x)}$ denote the mild solution to (sEP). Identify the correct statement(s), if any. **2 points**

- If $\frac{d\varphi^{(x)}}{dt}$ exists and is continuous on $[s, T]$, then $\varphi^{(x)}$ is a classical solution to (sEP)
- There is a constant $C > 0$ such that $\sup_{t \in [s, T]} \|\varphi^{(x)}(t) - \varphi^{(y)}(t)\| \leq C\|x - y\|$.
- If $f \in C^1$, then for each $x \in X$, $\varphi^{(x)}$ is a classical solution to (sEP).
- If $\varphi^{(x)}(t) \in D(A)$, for each $t \in [s, T]$, then $\varphi^{(x)}$ is a classical solution to (sEP).

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $\frac{d\varphi^{(x)}}{dt}$ exists and is continuous on $[s, T]$, then $\varphi^{(x)}$ is a classical solution to (sEP)
There is a constant $C > 0$ such that $\sup_{t \in [s, T]} \|\varphi^{(x)}(t) - \varphi^{(y)}(t)\| \leq C\|x - y\|$.
If $f \in C^1$, then for each $x \in X$, $\varphi^{(x)}$ is a classical solution to (sEP).
If $\varphi^{(x)}(t) \in D(A)$, for each $t \in [s, T]$, then $\varphi^{(x)}$ is a classical solution to (sEP).

5) Assume that A is the infinitesimal generator of a C_0 semigroup $\{T(t)\}_{t \geq 0}$ on X . Let $\varphi^{(x)}$ denote the mild solution to (sEP) and a function h be given by $h(t) = f(t, \varphi^{(x)}(t))$. If $\frac{d\psi}{dt} = A\psi(t) + h(t)$, $\psi(s) = x$ has a classical solution, then **2 points**

- $\psi(t) = T(t-s)x + \int_s^t T(t-r)h(r)dr$,
- $\psi(t) = \varphi^{(x)}(t)$ for each $t \in [s, T]$
- $\varphi^{(x)}$ is a classical solution to (sEP).
- If $\varphi^{(x)}(t) \in D(A)$, for each $t \in [s, T]$, then $\varphi^{(x)}$ is a classical solution to (sEP).

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\psi(t) = T(t-s)x + \int_s^t T(t-r)h(r)dr$,
 $\psi(t) = \varphi^{(x)}(t)$ for each $t \in [s, T]$
 $\varphi^{(x)}$ is a classical solution to (sEP).
If $\varphi^{(x)}(t) \in D(A)$, for each $t \in [s, T]$, then $\varphi^{(x)}$ is a classical solution to (sEP).