Unit 14 - Week 12

Assignment 12

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-04-22, 23:59 IST.

1) Identify the necessary conditions for existence of the mild solution on $[s,T]$ of the following inhomogeneous evolution problem *2 points*

$$\frac{d\varphi}{dt} = A(t)\varphi(t) + f(t)\ \forall\, t \in (s,T],\ \varphi(s) = x$$

- [ ] integrability of f
- [ ] continuous differentiability of f
- [ ] $D(A(t)) = X$ and $\|A(t)\|_{BL(X)} < \infty$ for all $t \in [0,T]$
- [ ] existence of an ES associated with $\{A(t)\}_{t\in[0,T]}$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
integrability of f
existence of an ES associated with $\{A(t)\}_{t\in[0,T]}$.

2) Assume that A is the infinitesimal generator of a C_0 semigroup $\{T(t)\}_{t\geq 0}$ on X. If $\varphi : [s,T] \to X$ is the mild solution to *2 points*

$$\frac{d\varphi(t)}{dt} = A\varphi(t) + f(t,\varphi(t)) \text{ for } t \in (s,T]\ \varphi(s) = x, \qquad (sEP)$$

identify the correct statement(s), if any.

- [ ] φ solves $\varphi(t) = T(t-s)x + \int_s^t T(t-r)f(r,\varphi(r))dr$
- [ ] $\varphi(t) = T(t-s)x + \int_s^t T(t-r)f(r,x)dr$
- [ ] $\varphi(t) = T(t-s)x$
- [ ] $\varphi(t) = T(t)x$

No, the answer is incorrect.
Score: 0

Accepted Answers:
φ solves $\varphi(t) = T(t-s)x + \int_s^t T(t-r)f(r,\varphi(r))dr$

3) Assume that A is the infinitesimal generator of a C_0 semigroup $\{T(t)\}_{t\geq 0}$ on X. Let φ be the continuous solution to $\varphi(t) = T(t-s)x + \int_s^t T(t-r)f(r,\varphi(r))dr$, *2 points*

then $\frac{d\varphi}{dt}$, if exists, is equal to

- [ ] $A\varphi(t) + f(t,\varphi(t))$
- [ ] $\frac{\partial}{\partial t}f(t,\varphi(t)) + \frac{\partial}{\partial v}f(s,v)|_{v=\varphi(t)}$
- [ ] w where w solves
$$w(t) = g(t) + \int_s^t T(t-r)\frac{\partial}{\partial v}f(s,v)|_{v=\varphi(r)}w(r)dr$$
$$g(t) = T(t-s)f(s,\varphi(s)) + AT(t-s)x + \int_s^t T(t-r)\frac{\partial}{\partial t}f(r,\varphi(r))dr.$$
- [ ] None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
$A\varphi(t) + f(t,\varphi(t))$
w where w solves
$$w(t) = g(t) + \int_s^t T(t-r)\frac{\partial}{\partial v}f(s,v)|_{v=\varphi(r)}w(r)dr$$
$$g(t) = T(t-s)f(s,\varphi(s)) + AT(t-s)x + \int_s^t T(t-r)\frac{\partial}{\partial t}f(r,\varphi(r))dr.$$

4) Let $[s,T]\times X \ni (t,x) \mapsto f(t,x) \in X$ be continuous in t and uniformly Lipschitz in x. Also assume that A is the infinitesimal generator of a C_0 semigroup $\{T(t)\}_{t\geq 0}$ on X. Let $\varphi^{(x)}$ denote the mild solution to (sEP). Identify the correct statement(s), if any. *2 points*

- [ ] If $\frac{d\varphi^{(x)}}{dt}$ exists and is continuous on $[s,T]$, then $\varphi^{(x)}$ is a classical solution to (sEP)
- [ ] There is a constant $C > 0$ such that $\sup_{t\in[s,T]} \|\varphi^{(x)}(t) - \varphi^{(y)}(t)\| \leq C\|x-y\|$.
- [ ] If $f \in C^1$, then for each $x \in X$, $\varphi^{(x)}$ is a classical solution to (sEP).
- [ ] If $\varphi^{(x)}(t) \in D(A)$, for each $t \in [s,T]$, then $\varphi^{(x)}$ is a classical solution to (sEP).

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $\frac{d\varphi^{(x)}}{dt}$ exists and is continuous on $[s,T]$, then $\varphi^{(x)}$ is a classical solution to (sEP)
There is a constant $C > 0$ such that $\sup_{t\in[s,T]} \|\varphi^{(x)}(t) - \varphi^{(y)}(t)\| \leq C\|x-y\|$.
If $f \in C^1$, then for each $x \in X$, $\varphi^{(x)}$ is a classical solution to (sEP).
If $\varphi^{(x)}(t) \in D(A)$, for each $t \in [s,T]$, then $\varphi^{(x)}$ is a classical solution to (sEP).

5) Assume that A is the infinitesimal generator of a C_0 semigroup $\{T(t)\}_{t\geq 0}$ on X. Let $\varphi^{(x)}$ denote the mild solution to (sEP) and a function h be given by $h(t) = f(t,\varphi^{(x)}(t))$. If $\frac{d\psi}{dt} = A\psi(t) + h(t), \psi(s) = x$ has a classical solution, then *2 points*

- [ ] $\psi(t) = T(t-s)x + \int_s^t T(t-r)h(r)dr$,.
- [ ] $\psi(t) = \varphi^{(x)}(t)$ for each $t \in [s,T]$
- [ ] $\varphi^{(x)}$ is a classical solution to (sEP).
- [ ] If $\varphi^{(x)}(t) \in D(A)$, for each $t \in [s,T]$, then $\varphi^{(x)}$ is a classical solution to (sEP).

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\psi(t) = T(t-s)x + \int_s^t T(t-r)h(r)dr$,.
$\psi(t) = \varphi^{(x)}(t)$ for each $t \in [s,T]$
$\varphi^{(x)}$ is a classical solution to (sEP).
If $\varphi^{(x)}(t) \in D(A)$, for each $t \in [s,T]$, then $\varphi^{(x)}$ is a classical solution to (sEP).