Unit 13 - Week 11

Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-04-15, 23:59 IST.

1) Let $\{T(t)\}_{t\geq 0}$ be a C_0 semigroup on a Banach space V and A being the infinitesimal generator of $\{T(t)\}_{t\geq 0}$. Fix $\lambda \in \rho(A)$. Three of the following operators are identical. Find out the one which is not identical to the others. ***2 points***

- $\lambda A(\lambda I - A)^{-1}$
- $\lambda^2(\lambda I - A)^{-1} - \lambda I$
- $\lambda A(\lambda I + A)$
- $\lambda A\left(\int_0^\infty e^{-t\lambda}T(t)dt\right)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\lambda A(\lambda I + A)$

2) Let A be a bounded linear invertible operator on V. Assume that v_1 and v_2 are in V and $v : [0, \infty) \to V$. Find out the statement which is not identical to the others. ***2 points***

- v solves $\frac{dv}{dt} = Av + v_1$ and $v(0) = v_2$
- v solves $\frac{dv}{dt} = Av + v_2$ and $v(0) = v_1$
- $v(t) = e^{tA}v_1 + \left(e^{tA} - I\right)A^{-1}v_2$
- $v(t) = e^{tA}\left(v_1 + \left(\int_0^t e^{-sA}ds\right)v_2\right)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
v solves $\frac{dv}{dt} = Av + v_1$ and $v(0) = v_2$

3) Let $\{T(t)\}_{t\geq 0}$ be a C_0 semigroup on a Banach space V and $f : [0, \infty) \to V$. Identify the correct condition on f so that $t \mapsto \int_0^t T(t-s)f(s)ds$ is differentiable. ***2 points***

- f is continuous
- f is integrable
- f is continuously differentiable
- None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
f is continuously differentiable

4) Let $\{T(t)\}_{t\geq 0}$ be a C_0 semigroup on a Banach space V and A being its infinitesimal generator. Set $U(t, s) := T(t - s)$ for every $t \geq s$. Find out the correct statement(s), if any. ***4 points***

- $[0, \infty) \ni t \mapsto T(t) \in BL(V)$ is differentiable at $t = 0$
- If $t \mapsto T(t)$ is differentiable at $t = 0$ it is differentiable at all $t > 0$
- $t \mapsto T(t)$ is continuous at all $t > 0$
- $\{U(t, s)\}_{0\leq s\leq t<\infty}$ gives an evolution system
- $U(t, s)$ is continuous in s and t variables
- $U(t, s)$ is differentiable in s and t variables
- If v is in the domain of definition of A, $U(t, s)v$ is differentiable in both the variables s and t
- $\frac{d}{dt}U(t, s)v = AU(t, s)v$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $t \mapsto T(t)$ is differentiable at $t = 0$ it is differentiable at all $t > 0$
$t \mapsto T(t)$ is continuous at all $t > 0$
$\{U(t, s)\}_{0\leq s\leq t<\infty}$ gives an evolution system
$U(t, s)$ is continuous in s and t variables
If v is in the domain of definition of A, $U(t, s)v$ is differentiable in both the variables s and t
$\frac{d}{dt}U(t, s)v = AU(t, s)v$