

Unit 13 - Week 11

Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

Week 3

Week 4

Week 5

Week 6

Week 7

Week 8

Week 9

Week 10

Week 11

- Mild solution to inhomogeneous initial value problem
- Sufficient condition for existence of classical solution of IVP
- Tutorial on Resolvent operator
- Feynman-Kac formula and the formula of variations of constants
- Non-autonomous evolution problem and mild/generalized solution

Quiz : Assignment 11

Probabilistic Methods in PDE: Week 11 Feedback form

Week 12

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Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-04-15, 23:59 IST.

1) Let $\{T(t)\}_{t \geq 0}$ be a C_0 semigroup on a Banach space V and A being the infinitesimal generator of $\{T(t)\}_{t \geq 0}$. Fix $\lambda \in \rho(A)$. Three of the following **2 points** operators are identical. Find out the one which is not identical to the others.

- $\lambda A(\lambda I - A)^{-1}$
- $\lambda^2(\lambda I - A)^{-1} - \lambda I$
- $\lambda A(\lambda I + A)$
- $\lambda A \left(\int_0^\infty e^{-t\lambda} T(t) dt \right)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\lambda A(\lambda I + A)$

2) Let A be a bounded linear invertible operator on V . Assume that v_1 and v_2 are in V and $v : [0, \infty) \rightarrow V$. Find out the statement which is not identical to the others. **2 points**

- v solves $\frac{dv}{dt} = Av + v_1$ and $v(0) = v_2$
- v solves $\frac{dv}{dt} = Av + v_2$ and $v(0) = v_1$
- $v(t) = e^{tA}v_1 + (e^{tA} - I)A^{-1}v_2$
- $v(t) = e^{tA} \left(v_1 + \left(\int_0^t e^{-sA} ds \right) v_2 \right)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 v solves $\frac{dv}{dt} = Av + v_1$ and $v(0) = v_2$

3) Let $\{T(t)\}_{t \geq 0}$ be a C_0 semigroup on a Banach space V and $f : [0, \infty) \rightarrow V$. Identify the correct condition on f so that $t \mapsto \int_0^t T(t-s)f(s)ds$ **2 points** is differentiable.

- f is continuous
- f is integrable
- f is continuously differentiable
- None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
 f is continuously differentiable

4) Let $\{T(t)\}_{t \geq 0}$ be a C_0 semigroup on a Banach space V and A being its infinitesimal generator. Set $U(t, s) := T(t-s)$ for every $t \geq s$. Find out the correct statement(s), if any. **4 points**

- $[0, \infty) \ni t \mapsto T(t) \in BL(V)$ is differentiable at $t = 0$
- If $t \mapsto T(t)$ is differentiable at $t = 0$ it is differentiable at all $t > 0$
- $t \mapsto T(t)$ is continuous at all $t > 0$
- $\{U(t, s)\}_{0 \leq s \leq t < \infty}$ gives an evolution system
- $U(t, s)$ is continuous in s and t variables
- $U(t, s)$ is differentiable in s and t variables
- If v is in the domain of definition of A , $U(t, s)v$ is differentiable in both the variables s and t
- $\frac{d}{dt}U(t, s)v = AU(t, s)v$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $t \mapsto T(t)$ is differentiable at $t = 0$ it is differentiable at all $t > 0$
 $t \mapsto T(t)$ is continuous at all $t > 0$
 $\{U(t, s)\}_{0 \leq s \leq t < \infty}$ gives an evolution system
 $U(t, s)$ is continuous in s and t variables
If v is in the domain of definition of A , $U(t, s)v$ is differentiable in both the variables s and t
 $\frac{d}{dt}U(t, s)v = AU(t, s)v$