Unit 12 - Week 10

Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

Week 3

Week 4

Week 5

Week 6

Week 7

Week 8

Week 9

Week 10

- Semigroup of bounded linear operators on Banach space part 2
- Growth property of C0 semigroup
- Unique semigroup generated by a bounded linear operator
- Homogeneous initial value problem
- Mild solution to homogeneous initial value problem
- Quiz : Assignment 10
- Probabilistic Methods in PDE: Week 10 Feedback form

Week 11

Week 12

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Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-04-08, 23:59 IST.

1) Let W denote the Brownian motion. *2 points*

Consider the following Cauchy problem

$$\left(\frac{\partial}{\partial t} + kx\frac{\partial}{\partial x} + \frac{\sigma^2}{2}x^2\frac{\partial^2}{\partial x^2} - k\right)v(t,x) = 0$$
$$v(T,x) = \max(0, x-c).$$

Find out the correct SDE so that the solution to this problem can be written as

$$v(t,x) = e^{-k(T-t)}E\left[\max(0, X_T^{(t,x)} - c)\right]$$

where $\{X_u^{(t,x)}\}_{u\in[t,T]}$ solves

- $dX_u = kdu + \sigma dW_u,\ X_t = x$
- $dX_u = kdu + \sigma^2 dW_u,\ X_t = x$
- $dX_u = kX_u du + \sigma X_u dW_u,\ X_t = x$
- $dX_u = kX_u du + \frac{\sigma^2}{2}X_u^2 dW_u,\ X_t = x$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$dX_u = kX_u du + \sigma X_u dW_u,\ X_t = x$

2) Consider the following Cauchy problem *2 points*

$$\left(\frac{\partial}{\partial t} + \mu x\frac{\partial}{\partial x} + \frac{\sigma^2}{2}x^2\frac{\partial^2}{\partial x^2} - k\right)v(t,x) = 0$$
$$v(T,x) = \max(0, x-c).$$

Find out the correct solution.

- $v(t,x) = e^{-\mu(T-t)}E\left[\max(0, X_T^{(t,x)} - c)\right]$ where $\{X_u^{(t,x)}\}_{u\in[t,T]}$ solves $dX_u = kX_u du + \sigma X_u dW_u,\ X_t = x$
- $v(t,x) = e^{-k(T-t)}E\left[\max(0, X_T^{(t,x)} - c)\right]$ where $\{X_u^{(t,x)}\}_{u\in[t,T]}$ solves $dX_u = \mu X_u du + \sigma X_u dW_u,\ X_t = x$
- $v(t,x) = e^{-\mu(T-t)}E\left[\max(0, X_T^{(t,x)} - c)\right]$ where $\{X_u^{(t,x)}\}_{u\in[t,T]}$ solves $dX_u = kdu + \sigma dW_u,\ X_t = x$
- $v(t,x) = e^{-k(T-t)}E\left[\max(0, X_T^{(t,x)} - c)\right]$ where $\{X_u^{(t,x)}\}_{u\in[t,T]}$ solves $dX_u = \mu du + \sigma dW_u,\ X_t = x$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$v(t,x) = e^{-k(T-t)}E\left[\max(0, X_T^{(t,x)} - c)\right]$ *where* $\{X_u^{(t,x)}\}_{u\in[t,T]}$ *solves*
$dX_u = \mu X_u du + \sigma X_u dW_u,\ X_t = x$

3) Let $X = \{X_t\}_{t\geq 0}$ be a homogeneous continuous time finite state Markov chain on $\{1,2,\ldots,n\}$ with rate matrix Λ. Identify the correct statements *2 points*

- $E(v(X_t)|X_0 = i) = (e^{t\Lambda}v)(i)$
- $t \mapsto e^{t\Lambda}$ is strongly continuous
- $t \mapsto e^{t\Lambda}$ is uniformly continuous
- $P(X_t = j|X_0 = i) = (e^{t\Lambda})_{i,j}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$E(v(X_t)|X_0 = i) = (e^{t\Lambda}v)(i)$
$t \mapsto e^{t\Lambda}$ *is strongly continuous*
$t \mapsto e^{t\Lambda}$ *is uniformly continuous*
$P(X_t = j|X_0 = i) = (e^{t\Lambda})_{i,j}$

4) Consider $V := \{f : \mathbb{R} \to \mathbb{R}|$ bounded continuous$\}$ with the sup norm. For $f \in V$ define $T(t)f(a) := f(a+ct)$ where $t \geq 0$ *4 points*

- $t \mapsto T(t)$ is strongly continuous
- $t \mapsto T(t)$ is uniformly continuous
- The generator of $\{T(t)\}_{t\geq 0}$
- The generator of $\{T(t)\}_{t\geq 0}$ is not a bounded linear operator
- $T(t)$ is a bounded linear operator for all $t \geq 0$
- For every $f \in V$ and every $t \geq 0$, $T(t)f$ is in the domain of the generator of the semigroup
- For every $f \in V$ and every $t \geq 0$, $\int_0^t T(s)f ds$ is in the domain of the generator of the semigroup.
- $\frac{d}{dt}T(t)f(a) = c\frac{d}{da}T(t)f(a) = T(t)c\frac{d}{da}f(a).$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$t \mapsto T(t)$ *is strongly continuous*
The generator of $\{T(t)\}_{t\geq 0}$ *is not a bounded linear operator*
$T(t)$ *is a bounded linear operator for all* $t \geq 0$
For every $f \in V$ *and every* $t \geq 0$, $\int_0^t T(s)f ds$ *is in the domain of the generator of the semigroup.*
$\frac{d}{dt}T(t)f(a) = c\frac{d}{da}T(t)f(a) = T(t)c\frac{d}{da}f(a).$