

Unit 11 - Week 9

Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

Week 3

Week 4

Week 5

Week 6

Week 7

Week 8

Week 9

Functional Stochastic Differential Equations

Statement of Dirichlet and Cauchy problems with variable coefficients elliptic operators

Cauchy Problem with variable coefficients: Feynman-Kac formula: Part 1

Cauchy Problem with variable coefficients: Feynman-Kac formula: Part 2

Semigroup of bounded linear operators on Banach space - Part 1

Quiz : Assignment 9

Probabilistic Methods in PDE: Week 9 Feedback form

Week 10

Week 11

Week 12

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Assignment 9

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2020-04-01, 23:59 IST.

Let W denote the Brownian motion.

Consider the following list of operators

(a) $\mu x \frac{\partial}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2}{\partial x^2}$,

(b) $\mu x \frac{\partial}{\partial x} + \frac{\sigma^2}{2} x^2 \frac{\partial^2}{\partial x^2}$,

(c) $(\mu - x) \frac{\partial}{\partial x} + \frac{\sigma^2}{2} x \frac{\partial^2}{\partial x^2}$,

(d) $(\mu - x) \frac{\partial}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2}{\partial x^2}$.

Match the correct operator associated to the following SDEs.

1) $dX_t = \mu X_t dt + \sigma X_t dW_t$.

1 point

- a
 b
 c
 d

No, the answer is incorrect. Score: 0

Accepted Answers: b

2) $dX_t = \mu X_t dt + \sigma dW_t$.

1 point

- a
 b
 c
 d

No, the answer is incorrect. Score: 0

Accepted Answers: a

3) $dX_t = (\mu - X_t) dt + \sigma dW_t$

1 point

- a
 b
 c
 d

No, the answer is incorrect. Score: 0

Accepted Answers: d

4) $dX_t = (\mu - X_t) dt + \sigma \sqrt{X_t} dW_t$

1 point

- a
 b
 c
 d

No, the answer is incorrect. Score: 0

Accepted Answers: c

5) Identify which of the following equations are nontrivial Functional Stochastic Differential Equations

2 points

$dX_t = (\int_0^t X_s ds) dt + X_t dW_t$

$dX_t = (\int_0^t X_s ds) dt + X_t dt$

$dX_t = X_t dt + (\int_0^t X_s ds) dW_t$

$dX_t = X_t dt + (\int_0^t \sin(s) ds) X_t dW_t$

$dX_t = (\int_0^t X_s ds) dt + (\int_0^t \sin(X_s) ds) dW_t$.

No, the answer is incorrect. Score: 0

Accepted Answers:

$dX_t = (\int_0^t X_s ds) dt + X_t dW_t$

$dX_t = X_t dt + (\int_0^t X_s ds) dW_t$

$dX_t = (\int_0^t X_s ds) dt + (\int_0^t \sin(X_s) ds) dW_t$.

6) Identify the correct statements.

2 points

If for each $x \in \mathbb{R}^d$, $a(x)$ is a symmetric positive definite matrix of order d , then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij}(x) \frac{\partial^2}{\partial x_i \partial x_j}$ is elliptic.

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If a is a constant symmetric positive definite matrix of order d , then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij} \frac{\partial^2}{\partial x_i \partial x_j}$ is uniformly elliptic.

If a is a constant symmetric positive definite matrix of order d , then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij} \|x\|^2 \frac{\partial^2}{\partial x_i \partial x_j}$ is uniformly elliptic.

No, the answer is incorrect. Score: 0

Accepted Answers:

If for each $x \in \mathbb{R}^d$, $a(x)$ is a symmetric positive definite matrix of order d , then the operator

$\sum_{i=1}^d \sum_{j=1}^d a_{ij}(x) \frac{\partial^2}{\partial x_i \partial x_j}$ is elliptic.

If a is a constant symmetric positive definite matrix of order d , then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij} \frac{\partial^2}{\partial x_i \partial x_j}$ is uniformly elliptic.

7) Fix a $T > 0$. Let $M_T^* := \max_{t \in [0, T]} M_t$, where M is a continuous local martingale. Find out the correct statements.

2 points

For each $m \geq 1$, there exists a $K_m > 0$ such that $E(W_T^*)^{2m} \leq K_m T^m$

For each $m \geq 1$, $E(W_T^*)^{2m} < \infty$.

Let $M_t := \exp(W_t - \frac{t}{2})$, then for each $m \geq 1$, there exists a $K_m > 0$ such that $E(M_T^*)^{2m} \leq K_m E[(\int_0^T M_t^2 dt)^m]$

Let $M_t := M_0 \exp(W_t - \frac{t}{2})$, where M_0 is a standard Cauchy distribution. Then $E(M_T^*)^{2m} < \infty$

No, the answer is incorrect. Score: 0

Accepted Answers:

For each $m \geq 1$, there exists a $K_m > 0$ such that $E(W_T^*)^{2m} \leq K_m T^m$

For each $m \geq 1$, $E(W_T^*)^{2m} < \infty$.

Let $M_t := \exp(W_t - \frac{t}{2})$, then for each $m \geq 1$, there exists a $K_m > 0$ such that

$E(M_T^*)^{2m} \leq K_m E[(\int_0^T M_t^2 dt)^m]$