Unit 11 - Week 9

Assignment 9

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-04-01, 23:59 IST.

Let W denote the Brownian motion.

Consider the following list of operators
(a) $\mu x\frac{\partial}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2}$,
(b) $\mu x\frac{\partial}{\partial x} + \frac{\sigma^2}{2}x^2\frac{\partial^2}{\partial x^2}$,
(c) $(\mu - x)\frac{\partial}{\partial x} + \frac{\sigma^2}{2}x\frac{\partial^2}{\partial x^2}$,
(d) $(\mu - x)\frac{\partial}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2}$.
Match the correct operator associated to the following SDEs.

1) $dX_t = \mu X_t dt + \sigma X_t dW_t$. *1 point*

- a
- b
- c
- d

No, the answer is incorrect.
Score: 0
Accepted Answers:
b

2) $dX_t = \mu X_t dt + \sigma dW_t$. *1 point*

- a
- b
- c
- d

No, the answer is incorrect.
Score: 0
Accepted Answers:
a

3) $dX_t = (\mu - X_t)dt + \sigma dW_t$ *1 point*

- a
- b
- c
- d

No, the answer is incorrect.
Score: 0
Accepted Answers:
d

4) $dX_t = (\mu - X_t)dt + \sigma\sqrt{X_t}dW_t$ *1 point*

- a
- b
- c
- d

No, the answer is incorrect.
Score: 0
Accepted Answers:
c

5) Identify which of the following equations are nontrivial Functional Stochastic Differential Equations *2 points*

- $dX_t = (\int_0^t X_s ds)dt + X_t dW_t$
- $dX_t = (\int_0^t X_s ds)dt + X_t dt$
- $dX_t = X_t dt + (\int_0^t X_s ds)dW_t$
- $dX_t = X_t dt + (\int_0^t \sin(s)ds)X_t dW_t$
- $dX_t = (\int_0^t X_s ds)dt + (\int_0^t \sin(X_s)ds)dW_t$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$dX_t = (\int_0^t X_s ds)dt + X_t dW_t$
$dX_t = X_t dt + (\int_0^t X_s ds)dW_t$
$dX_t = (\int_0^t X_s ds)dt + (\int_0^t \sin(X_s)ds)dW_t$.

6) Identify the correct statements. *2 points*

- If for each $x \in \mathbb{R}^d$, $a(x)$ is a symmetric positive definite matrix of order d, then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j}$ is elliptic.
- If for each $x \in \mathbb{R}^d$, $a(x)$ is a symmetric positive definite matrix of order d, then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j}$ is uniformly elliptic.
- If a is a constant symmetric positive definite matrix of order d, then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij}\frac{\partial^2}{\partial x_i \partial x_j}$ is uniformly elliptic.
- If a is a constant symmetric positive definite matrix of order d, then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij}\|x\|^2\frac{\partial^2}{\partial x_i \partial x_j}$ is uniformly elliptic.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If for each $x \in \mathbb{R}^d$, $a(x)$ is a symmetric positive definite matrix of order d, then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j}$ is elliptic.
If a is a constant symmetric positive definite matrix of order d, then the operator $\sum_{i=1}^d \sum_{j=1}^d a_{ij}\frac{\partial^2}{\partial x_i \partial x_j}$ is uniformly elliptic.

7) Fix a $T > 0$. Let $M_T^* := \max_{t\in[0,T]} M_t$, where M is a continuous local martingale. Find out the correct statements. *2 points*

- For each $m \geq 1$, there exists a $K_m > 0$ such that $E(W_T^*)^{2m} \leq K_m T^m$
- For each $m \geq 1$, $E(W_T^*)^{2m} < \infty$.
- Let $M_t := \exp(W_t - \frac{t}{2})$, then for each $m \geq 1$, there exists a $K_m > 0$ such that $E(M_T^*)^{2m} \leq K_m E[(\int_0^T M_t^2 dt)^m]$
- Let $M_t := M_0 \exp(W_t - \frac{t}{2})$, where M_0 is a standard Cauchy distribution. Then $E(M_T^*)^{2m} < \infty$

No, the answer is incorrect.
Score: 0
Accepted Answers:
For each $m \geq 1$, there exists a $K_m > 0$ such that $E(W_T^*)^{2m} \leq K_m T^m$
For each $m \geq 1$, $E(W_T^*)^{2m} < \infty$.
Let $M_t := \exp(W_t - \frac{t}{2})$, then for each $m \geq 1$, there exists a $K_m > 0$ such that $E(M_T^*)^{2m} \leq K_m E[(\int_0^T M_t^2 dt)^m]$