Assignment 5

Due on 2020-09-04, 23:59 IST.

Note: All rings considered in this course are commutative with unity.

1. Choose the correct statements. Z denotes the ring of integers, R denotes the field of real numbers.

   a) Let $f(x) = Z[x]$ be a polynomial of degree 2. Suppose $f(x)$ has two distinct roots in $Z$. Then the ideal generated by $f(x)$ in $Z[x]$ is contained in exactly two maximal ideals of $Z[x]$.
   b) Let $f(x) = Z[x]$ be a polynomial of degree 2. Suppose $f(x)$ has two distinct roots in $Z$. Then the ideal generated by $f(x)$ in $Z[x]$ is contained in infinitely many maximal ideals of $Z[x]$.
   c) Let $f(x) = R[x]$ be a polynomial of degree 2. Then the ideal generated by $f(x)$ in $R[x]$ is contained in exactly two maximal ideals of $R[x]$.
   d) Let $f(x) = R[x]$ be a polynomial of degree 2. Then the ideal generated by $f(x)$ in $R[x]$ is contained in at most two maximal ideals of $R[x]$.

   No, the answer is incorrect.
   Accepted answers:
   a) True
   b) False
   c) True
   d) True

2. Which of the following rings $R$ are PIDs?

   a) $R$ is any integral domain.
   b) $R = K[x]$, where $K$ is a field.
   c) $R = K[x, y]$, where $K$ is a field.

   No, the answer is incorrect.
   Accepted answers:
   a) False
   b) True
   c) True

3. Which of the following rings $R$ are UFDs? $Z$ denotes the ring of integers and $i$ denotes a complex square root of $-1$.

   a) $R$ is any integral domain.
   b) $R = Z[i]$
   c) $R = Z[\sqrt{-5}]$
   d) $R = Z[x]$

   No, the answer is incorrect.
   Accepted answers:
   a) False
   b) True
   c) False
   d) True

4. Which of the given elements are irreducible in the given rings? Here $Z$ is the ring of integers and $Q$ is the field of rational numbers.

   $\sqrt{2} \in Z$
   $\sqrt{3} \in Z[x]$
   $2 + i \in Q(i)$
   $2 + i \in Z[i]$

   No, the answer is incorrect.
   Accepted answers:
   $\sqrt{2}$ is irreducible.
   $\sqrt{3}$ is irreducible.
   $2 + i$ is irreducible.
   $2 + i$ is irreducible.

5. Choose the correct statements.

   a) If $R$ is a commutative ring, then $R[x]$ is also a commutative ring.
   b) If $R$ is a PID and $S$ is a subring of $R$, then $S$ is a PID.
   c) If $R$ is a UFD, then $R$ is a UFD.
   d) If $R$ is a PDR, then $R$ is a PID.

   No, the answer is incorrect.
   Accepted answers:
   a) True
   b) True
   c) False
   d) False

6. Choose the correct statements. $Z$ denotes the ring of integers; $Q$ and $Q$ denote the fields of rational and complex numbers, respectively.

   a) A greatest common divisor of $4 + 2i$ in $Z[i]$ is $1$.
   b) A greatest common divisor of $2 + i$ and $2i + 1$ in $Z[i]$ is $1 + i$.
   c) A greatest common divisor of $2 + i$ and $2i - 10x + 3y + 15z - 5 i Q(x, y, z)$.
   d) A greatest common divisor of $x + 2i - 7y + 2z + 11x - 10z \in C(x, y, z)$.

   No, the answer is incorrect.
   Accepted answers:
   a) False
   b) False
   c) False
   d) False