**Assignment 3**

The due date for submitting this assignment has passed. As our records you have not submitted this assignment.

Note: All rings considered in this course are commutative with unity.

1. Let $R$, $S$ be rings and let $J \subseteq R$ and $J \subseteq S$ be ideals. Let $\phi : R \rightarrow S$ be a ring homomorphism. Choose all the correct statements.

   - $\phi^{-1}(J)$ is always an ideal of $R$.
   - If $\phi$ is onto, then $\phi(J)$ is an ideal of $S$.
   - If the kernel of $\phi$ is contained in $J$, then $\phi(J)$ is an ideal of $S$.
   - If $\phi(J)$ is an ideal of $R$, then $J$ is contained in the kernel of $\phi$.

   No, the answer is incorrect.
   **Score 0**

   **Answered:**

   - $\phi^{-1}(J)$ is always an ideal of $R$.
   - If $\phi$ is onto, then $\phi(J)$ is an ideal of $S$.
   - If the kernel of $\phi$ is contained in $J$, then $\phi(J)$ is an ideal of $S$.
   - If $\phi(J)$ is an ideal of $R$, then $J$ is contained in the kernel of $\phi$.

2. Choose all the correct statements. $R, S$ denote the fields of real and complex numbers, respectively.

   - There exists an injective ring homomorphism $\mathbb{R} \rightarrow \mathbb{C}$.
   - There exists a ring homomorphism $\mathbb{C} \rightarrow \mathbb{R}$.
   - There exists a surjective ring homomorphism $\mathbb{R} \rightarrow \mathbb{C}$.
   - There exists a ring homomorphism $\mathbb{C} \rightarrow \mathbb{R}$.

   No, the answer is incorrect.
   **Score 0**

   **Answered:**

   - There exists an injective ring homomorphism $\mathbb{R} \rightarrow \mathbb{C}$.
   - There exists a ring homomorphism $\mathbb{C} \rightarrow \mathbb{R}$.
   - There exists a surjective ring homomorphism $\mathbb{R} \rightarrow \mathbb{C}$.
   - There exists a ring homomorphism $\mathbb{C} \rightarrow \mathbb{R}$.

3. Choose all the correct answers. $Z$ denotes the ring of integers.

   - The number of ideals in the quotient ring $\mathbb{Z}/6\mathbb{Z}$ is 3.
   - The number of ideals in the quotient ring $\mathbb{Z}/8\mathbb{Z}$ is 3.
   - The number of prime ideals in the quotient ring $\mathbb{Z}/7\mathbb{Z}$ is 1.
   - The number of maximal ideals in the quotient ring $\mathbb{Z}/7\mathbb{Z}$ is 3.

   No, the answer is incorrect.
   **Score 0**

   **Answered:**

   - The number of ideals in the quotient ring $\mathbb{Z}/6\mathbb{Z}$ is 3.
   - The number of ideals in the quotient ring $\mathbb{Z}/8\mathbb{Z}$ is 3.
   - The number of prime ideals in the quotient ring $\mathbb{Z}/7\mathbb{Z}$ is 1.
   - The number of maximal ideals in the quotient ring $\mathbb{Z}/7\mathbb{Z}$ is 3.

4. Let $R$ be an arbitrary ring. Choose all the correct statements.

   - If $I$ is a prime ideal of $R$ and $I \subseteq J$ is proper, then $J = I$.
   - If $I$ is a prime ideal of $R$ then the quotient ring $R/I$ does not contain any zero divisor.
   - If $I$ is a prime ideal of $R$ then $R/I$ is a field.
   - If $J$ is a prime ideal of $R$, then $I \subseteq J$.

   No, the answer is incorrect.
   **Score 0**

   **Answered:**

   - If $I$ is a prime ideal of $R$ and $I \subseteq J$ is proper, then $J = I$.
   - If $I$ is a prime ideal of $R$ then the quotient ring $R/I$ does not contain any zero divisor.
   - If $I$ is a prime ideal of $R$ then $R/I$ is a field.
   - If $J$ is a prime ideal of $R$, then $I \subseteq J$.

5. Choose all the correct statements. $Z$ denotes the ring of integers, $Q, R$, and $C$ denote the fields of rational, real, and complex numbers, respectively.

   - The ideal generated by $x^2 + 1$ is maximal in $\mathbb{Z}[x]$.
   - The ideal generated by $x^2 + 1$ is maximal in $Q[x]$.
   - The ideal generated by $x^2 + 1$ is maximal in $R[x]$.
   - The ideal generated by $x^2 + 1$ is maximal in $C[x]$.

   No, the answer is incorrect.
   **Score 0**

   **Answered:**

   - The ideal generated by $x^2 + 1$ is maximal in $\mathbb{Z}[x]$.
   - The ideal generated by $x^2 + 1$ is maximal in $Q[x]$.
   - The ideal generated by $x^2 + 1$ is maximal in $R[x]$.
   - The ideal generated by $x^2 + 1$ is maximal in $C[x]$.

6. Let $R$ be any nonzero ring. Choose all the correct statements. $Z$ denotes the ring of integers.

   - Suppose every proper ideal of $R$ is prime, then $R$ is a field.
   - Every nonzero proper ideal of $Z$ is maximal.
   - Every proper ideal of $Z$ is prime.
   - Every nonzero prime ideal of $Z$ is maximal.

   No, the answer is incorrect.
   **Score 0**

   **Answered:**

   - Suppose every proper ideal of $R$ is prime, then $R$ is a field.
   - Every nonzero proper ideal of $Z$ is maximal.
   - Every proper ideal of $Z$ is prime.
   - Every nonzero prime ideal of $Z$ is maximal.