Assignment Sheet - 11

1. Use Laplace transform to solve the initial boundary value problem

\[
\frac{\partial^2 u}{\partial x \partial t} = -k \cos kt, \ t > 0, x > 0, k \in \mathbb{R},
\]

\[u(x, 0) = x, \]

\[u(0, t) = 0.\]

2. Use Laplace transform to solve

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 4u, \ t > 0,
\]

\[u(0, t) = 0, \]

\[u(\pi, t) = 0.\]

3. Use Laplace transform to find the temperature \(u(x, t)\) that is governed by

\[
\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, t > 0,
\]

\[u(x, 0) = u_0, 0 < x < L, \]

\[u(L, t) = u_1, t > 0, \]

\[\frac{\partial u(0, t)}{\partial x} = 0, t > 0.\]

4. Use Laplace transform to find the solution of

\[f(x) = x + 2 \int_0^x \cos(x - t)f(t)dt.\]

5. Use Laplace transform to evaluate the integral

\[
\int_{-\infty}^{\infty} \frac{x \sin xt}{x^2 + a^2} \, dx, a > 0, t > 0.
\]