NUMERICAL ANALYSIS
Assignment -7 (week 7)
Total Marks - 25
Posted on - 4/9/2017 (Monday);
To be submitted on or before-13/9/2017 (Wednesday), 23.59 hours.

Problems on

• Finite difference methods

• Shooting Method

INSTRUCTIONS

• This is a question paper cum answer booklet.

• Take a print out of this.

• Present the details of the computations of the solution of each problem which you will have to show in the space provided at the bottom of the page.

• Fill in the answers in the space provided below each question.

• Scan the booklet and submit it as a pdf file before the deadline for evaluation.
1. Consider the BVP \( y'' = 4(y - x), \ 0 \leq x \leq 1, \ y(0) = 0, \ y(1) = 2. \) Use the linear shooting method to approximate the solution at \( x = 0.25, \ 1, \) and \( 0.75 \) (Recall this problem has been solved in practice problems 7 and solution has already been obtained at \( x = 0.5 \). Proceed in the same manner and find solutions at \( x = 0.25 \) and \( x = 0.75 \). You are dividing \([0, 1]\) into 4 equal parts of width \( h = \frac{1 - 0}{4} = 0.25 \). The \( x_i \)'s are \( x_0 = 0, \ x_1 = 0.25, \ x_2 = 0.5, \ x_3 = 0.75 \) and \( x_4 = 1 \). We already have solutions at \( x = 0.5 \). Determine \( y(0.25) \) and \( y(0.75) \). Present the results as below.

Fill in the blanks in Table: \((5 + 5 = 10 \text{ marks})\)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>( y_1 = y(0.25) = )</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( y_2 = y(0.5) = 0.8264 )</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>( y_3 = y(0.75) = )</td>
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</tbody>
</table>

Show your work for the solution of problem 1 in the space provided below.
2. Consider the BVP $y'' = y' + 2y + \cos x, \ 0 \leq x \leq \frac{\pi}{2}, \ y(0) = -0.3, \ y\left(\frac{\pi}{2}\right) = -0.1$. You want to solve this linear BVP using linear shooting method. You will consider two IVP’s and solve them first and then use their solution to obtain the solution of the BVP.
   (a) Formulate the first IVP.
   (b) formulate the second IVP
   (c) Then the solution of the given BVP in terms of the solutions of the two IVP’s in (a) & (b) is ______________.
You don’t have to solve the problem. Write down the answers to questions 2(a), (b) and (c). (5 marks)

Show your work for the solution of problem 2 in the space provided below.
3. consider the BVP \( y'' = 4(y - x), \ 0 \leq x \leq 1, \ y(0) = 0, \ y(1) = 2. \)

Use finite-difference method and solve the BVP by taking step-size \( h = \frac{1}{4}. \) Formulate the problem as a system \( A\omega = b \) where \( A \) is a \( 5 \times 5 \) matrix, \( \omega \) is a \( 5 \times 1 \) vector and \( b \) is a \( 5 \times 1 \) vector. You need not show the solution. (5 marks)

Show your work for the solution of problem 3 in the space provided below.
4. Consider $y'' + y = 1; \ y(0) = 1, \ y\left(\frac{\pi}{2}\right) = 0$. Use finite-difference method to solve the above BVP by taking step size $h = \frac{\pi}{8}$ and formulate the problem as a problem of solving a system of equations $A\omega = b$ where $\omega_i$ is an approximation to $y$ at $x_i$. (5 marks)

Show your work for the solution of problem 4 in the space provided below.