

Assignment-4

1. Find the singular points and classify them for the equation $x^3(x-2)y'' + x^3y' + 6y = 0$.
2. Find the power series solution for the equation $y'' + xy' + (1+x^2)y = 0$.
3. Solve the initial value problem $(x^2-1)y'' + 3xy' + xy = 0, y(0) = 4, y'(0) = 6$.
4. Find the power series solution around the point $x_0 = 1$ for the equation $x^2y'' + xy' + y = 0$.
5. Find a polynomial approximation of fourth degree to the solution of the equation $(1+2x)y'' - y' + y = 0, y(0) = 0, y'(0) = 1$.
6. Find a polynomial approximation of fourth degree to the solution of the equation $y'' + xy' + (1+x)y = 0, y(0) = -1, y'(0) = 0$.
7. Express the polynomials x^3 and $x^3 + 2x^2 - 3x + 6$ in terms of Legendre polynomials.
8. Use first recurrence relation for Legendre polynomials to show that the value of $\frac{P_{500}(L)}{P_{502}(L)}$ is negative when $P_{501}(L) = 0$.
9. Use first recurrence relation for Legendre polynomials to show that $\int_{-1}^1 x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2-1}$.
10. Use first recurrence relation for Legendre polynomials to find the value of $\int_{-1}^1 x^2 P_n^2(x) dx$.