

Unit 3 - Week 01

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Week 01
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Assignment 1

The due date for submitting this assignment has passed. **Due on 2019-08-14, 23:59 IST.**
 As per our records you have not submitted this assignment.

1) The adjoint equation of the differential equation $x^2 \frac{d^2 y}{dx^2} + (2x^3 + 1) \frac{dy}{dx} + y = 0$ is **1 point**

- a. $x^2 \frac{d^2 y}{dx^2} + (2x^3 - 1) \frac{dy}{dx} + 6x^2 y = 0$
- b. $x^2 \frac{d^2 y}{dx^2} + (4x - 2x^3 - 1) \frac{dy}{dx} + (3 - 6x^2) y = 0$
- c. $x^2 \frac{d^2 y}{dx^2} + (4x - 1) \frac{dy}{dx} + 6x^2 y = 0$
- d. $\frac{d^2 y}{dx^2} + (4x - 1) \frac{dy}{dx} + (1 - 2x^2) y = 0$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: b

2) The self-adjoint equation of the differential equation $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$ is **1 point**

HINT: The given differential equation is not a self adjoint equation. Transform it into an equivalent self adjoint equation.

- a. $\frac{d}{dx} \left[\frac{1}{x^2} \frac{dy}{dx} \right] + \frac{2}{x} y = 0$
- b. $\frac{d}{dx} \left[\frac{1}{x^2} \frac{dy}{dx} \right] + \frac{1}{x} y = 0$
- c. $\frac{d}{dx} \left[\frac{1}{x} \frac{dy}{dx} \right] + \frac{1}{x^2} y = 0$
- d. $\frac{d}{dx} \left[x^2 \frac{dy}{dx} \right] + \frac{2}{x^2} y = 0$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: a

3) The self-adjoint of the ODE $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$ is of the form $\frac{d}{dx} [p(x)y'] + q(x)y = 0$, then the values of $p(x)$ and $q(x)$ are **1 point**

- a. $p(x) = x, q(x) = x$
- b. $p(x) = \frac{1}{x}, q(x) = \frac{1}{x}$
- c. $p(x) = x, q(x) = \frac{1}{x}$
- d. $p(x) = \frac{1}{x}, q(x) = x$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: c

4) The orthonormal set of the orthogonal set of functions $\left\{ \sin\left(\frac{n\pi x}{L}\right) \right\}, n = 1, 2, 3, \dots$ on the interval $(0, L)$ is given by **1 point**

- a. $\left(\frac{2}{L}\right) \cos\left(\frac{n\pi x}{L}\right), n = 1, 2, 3, \dots$
- b. $\left(\frac{2}{L}\right)^{\frac{1}{2}} \cos\left(\frac{n\pi x}{L}\right), n = 1, 2, 3, \dots$
- c. $\left(\frac{2}{L}\right) \sin\left(\frac{n\pi x}{2L}\right), n = 1, 2, 3, \dots$
- d. $\left(\frac{2}{L}\right)^{\frac{1}{2}} \sin\left(\frac{n\pi x}{L}\right), n = 1, 2, 3, \dots$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: d

5) The eigenvalue of the Sturm-Liouville problem $y'' + \lambda y = 0, 0 \leq x \leq \pi, y(0) = 0, y(\pi) = 0$ are **1 point**

- a. $\frac{(n-1)^2}{2}$
- b. $\frac{(2n-1)^2 \pi^2}{4}$
- c. $\frac{n^2 \pi^2}{4}$
- d. $\frac{(2n-1)^2}{4}$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: d

6) The eigenvalues for the boundary value problem $y'' + \lambda y = 0, 0 \leq x \leq \pi, y(0) = 0, y(\pi) + y'(\pi) = 0$ satisfy **1 point**

- a. $\sqrt{\lambda} + \tan(\pi\lambda)$
- b. $\lambda + \tan(\pi\sqrt{\lambda})$
- c. $\sqrt{\lambda} + \tan(\pi\sqrt{\lambda})$
- d. $\lambda + \tan(\pi\lambda)$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: c

7) The eigen values of the Sturm-Liouville problem $\frac{d}{dx} [xy'] + \frac{\lambda}{x} y = 0, y'(1) = 0, y(b) = 0 (b > 1)$ are given by **1 point**

- a. $\frac{2n\pi}{\log b}$
- b. $\frac{(2n-1)\pi}{2 \log b}$
- c. $\frac{(2n-1)}{\log b}$
- d. $\frac{n\pi}{2 \log b}$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: b

8) The eigenfunctions of the Sturm-Liouville problem $y'' + \lambda y = 0 (\lambda > 0)$ with $y(0) = 0$ and $y'(L) = 0$ is equal to **1 point**

- a. $\sin\left(\frac{(2n-1)\pi}{2L} x\right)$
- b. $\cos\left(\frac{(2n-1)\pi}{2L} x\right)$
- c. $\sin\left(\frac{n\pi}{2L} x\right)$
- d. $\cos\left(\frac{n\pi}{2L} x\right)$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: a

9) The eigenfunctions of the Sturm-Liouville problem $4y'' - 4y' + (1 + \lambda)y = 0$ with $y(0) = 0, y(1) = 0$ is equal to **1 point**

- a. $e^{\frac{x}{2}} \cos(n\pi x)$
- b. $e^{-\frac{x}{2}} \sin(n\pi x)$
- c. $e^{\frac{x}{2}} \sin(n\pi x)$
- d. $e^{-\frac{x}{2}} \cos(n\pi x)$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: c

10) The Eigen vectors of the Sturm-Liouville problem $y'' + \lambda^2 y = 0, 0 < x < \pi$ with $y'(0) = y'(\pi) = 0$ are **1 point**

- a. $\sin(n x)$
- b. $\cos(n x)$
- c. $\sin\left(n + \frac{1}{2}\right) x$
- d. $\cos\left(n + \frac{1}{2}\right) x$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: b

11) The eigenvalues and eigen functions of the Sturm-Liouville problem $\frac{d}{dx} (xy') + \frac{\lambda}{x} y = 0$ with $y'(1) = 0 = y'(e^{2\pi})$ are **1 point**

- a. $y(x) = 1$ for $\lambda = 0$ and $y_n(x) = \cos(\sqrt{n} \log x)$ for $\lambda_n = n, n = 1, 2, 3, \dots$
- b. $y(x) = 1$ for $\lambda = 0$ and $y_n(x) = \cos(n \log x)$ for $\lambda_n = n^2, n = 1, 2, 3, \dots$
- c. $y(x) = 1$ for $\lambda = 0$ and $y_n(x) = \sin\left(\frac{n}{2} \log x\right)$ for $\lambda_n = \frac{n^2}{4}, n = 1, 2, 3, \dots$
- d. $y(x) = 1$ for $\lambda = 0$ and $y_n(x) = \cos\left(\frac{n}{2} \log x\right)$ for $\lambda_n = \frac{n^2}{4}, n = 1, 2, 3, \dots$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: d

12) The eigen functions of $\frac{d}{dx} (x^3 y') + \lambda xy = 0 (\lambda > 1), y(1) = 0 = y(e)$ are **1 point**

- a. $\frac{1}{x} \sin(n\pi \log x), n = 1, 2, 3, \dots$
- b. $\frac{1}{x} \sin(n \log x), n = 1, 2, 3, \dots$
- c. $\sin(n\pi \log x), n = 1, 2, 3, \dots$
- d. $x \sin(n\pi \log x), n = 1, 2, 3, \dots$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: a

13) The eigenvalues and eigen functions of the Sturm-Liouville problem $y'' + \lambda y = 0$ with $y(0) + y'(0) = 0$ and $y(1) + y'(1) = 0$ are **1 point**

- a. $y(x) = e^{-x}$ for $\lambda = -1$ and $y_n(x) = \sin(n\pi x) - \cos(n\pi x)$ for $\lambda_n = n\pi, n = 1, 2, \dots$
- b. $y(x) = e^{-x}$ for $\lambda = -1$ and $y_n(x) = \sin(n\pi x) - n\pi \cos(n\pi x)$ for $\lambda_n = n^2 \pi^2, n = 1, 2, \dots$
- c. $y(x) = e^{-x}$ for $\lambda = -1$ and $y_n(x) = \cos(n\pi x) - n\pi \sin(n\pi x)$ for $\lambda_n = n^2 \pi^2, n = 1, 2, \dots$
- d. $y(x) = e^{-x}$ for $\lambda = -1$ and $y_n(x) = \cos(n\pi x) - \sin(n\pi x)$ for $\lambda_n = n\pi, n = 1, 2, 3, \dots$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: b

14) The solution of the non-homogeneous Sturm-Liouville problem $-y'' = x, 0 < x < \pi$ with $y(0) = y(\pi) = 0$ is **1 point**

- a. $2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \sin(n x)$
- b. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \sin(n x)$
- c. $2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \cos(n x)$
- d. $2 \sum_{n=1}^{\infty} \frac{1}{n^3} \sin(n x)$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: a

15) The solution of the non-homogeneous Sturm-Liouville problem $-y'' = x, 0 < x < 1$ with $y(0) = y(1) = 0$ is **1 point**

- a. $\sum_{n=1}^{\infty} \frac{1}{n^3 \pi^3} \sin(n \pi x)$
- b. $2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3 \pi^3} \sin(n \pi x)$
- c. $\sum_{n=1}^{\infty} \frac{1}{n^3 \pi^3} \sin(n \pi x)$
- d. $2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 \pi^3} \cos(n \pi x)$

- a
- b
- c
- d

No, the answer is incorrect.
 Score: 0

Accepted Answers: b