Unit 3 - Week 1
Assignment 1

1. Let the function $f(x) = ax^2 + bx + c$ be differentiable. If $f(1) = 2$, $f(2) = 5$, and $f'(1) = 3$, find the values of $a$, $b$, and $c$.

2. Find the value of $k$ if $y = x^3 - 2x^2 + kx + 1$ is tangent to the line $y = 3x + 4$ at $x = 1$.

3. For the function $f(x) = x^3 - 3x^2 + kx$, what must be true about $k$ for the graph to have an inflection point at $(0, 0)$?

4. Suppose a manufacturer produces a certain product with a cost function $C(x) = 2x^2 + 3x + 10$. The company sets a price that guarantees a profit of $P(x) = 4x - x^2$. Determine the break-even points.

5. At what points does the function $f(x) = x^3 - 6x^2 + 9x + 4$ have horizontal tangents?

6. Let $f(x) = x^3 - 3x^2 + 2x - 1$. Find the minimum and maximum values of $f(x)$ on the interval $[0, 3]$.

7. For the function $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$, determine the intervals where the function is increasing.

8. For the function $f(x) = x^3 - 3x^2 + 2$, find the point where the tangent line has a slope of $-1$.

9. The position of a particle is given by $s(t) = t^4 - 4t^3 + 6t^2 - 3t + 5$ for $0 \leq t \leq 5$. Find the velocity and acceleration of the particle at $t = 2$.

10. If the profit function for a company is $P(x) = -2x^2 + 12x - 18$, where $x$ is the number of units produced, find the production level that maximizes profit.

11. For the function $f(x) = x^3 - 6x^2 + 9x + 4$, find the concavity of the graph on the interval $(-1, 3)$.

12. Find the local maximum and minimum values of the function $f(x) = x^3 - 3x^2 + 2x - 1$ on the interval $[-1, 3]$.

13. Suppose a company's revenue function is $R(x) = 5x^2 - x^3$ where $x$ is the number of units sold. If the company wants to maximize revenue, find the number of units they should produce.

14. The demand function for a product is $P(x) = 100 - 2x$, where $x$ is the quantity demanded. If the company's cost function is $C(x) = 10x + 50$, find the quantity that maximizes profit.

15. For the function $f(x) = x^3 - 3x^2 + 2x - 1$, find the point where the tangent line has a slope of $-1$.

16. If the profit function for a company is $P(x) = -2x^2 + 12x - 18$, where $x$ is the number of units produced, find the production level that maximizes profit.

17. For the function $f(x) = x^3 - 6x^2 + 9x + 4$, find the concavity of the graph on the interval $(-1, 3)$.

18. The position of a particle is given by $s(t) = t^4 - 4t^3 + 6t^2 - 3t + 5$ for $0 \leq t \leq 5$. Find the velocity and acceleration of the particle at $t = 2$.

19. Suppose a company's revenue function is $R(x) = 5x^2 - x^3$ where $x$ is the quantity demanded. If the company wants to maximize revenue, find the number of units they should produce.

20. The demand function for a product is $P(x) = 100 - 2x$, where $x$ is the quantity demanded. If the company's cost function is $C(x) = 10x + 50$, find the quantity that maximizes profit.