Unit 10 - Week 9

Week 9:
The due date for submitting this assignment has passed. Due on 2017-09-27, 23:59 IST.

Submitted assignment

Week 9

1) Find the optimal value by grid search method in between [(0,0), (2,2)]. Consider a step length as 0.25

\[ \text{minimize } f(x, y) = x - y + 2x^2 + y^2 + 2xy \]

a) \( f_{\min} = -1.25 \)

b) \( f_{\min} = -1.15 \)

c) \( f_{\min} = -1.875 \)

d) \( f_{\min} = -0.25 \)

No, the answer is incorrect.
Score: 0

Accepted Answers:

2) Solve the problem with direct substitution method

\[ \text{maximize } 50x + 2x^2 - 3y^2 - xy + 95y \]
\[ \text{s.t. } x + y = 25 \]

a) 800

b) 1000

c) 900

d) 1100

No, the answer is incorrect.
Score: 0

Accepted Answers:
3) Find the value of $x, y$ with direct substitution method

\[
\begin{align*}
\text{maximize } & 4x^2 + 5y^2 \\
\text{s.t. } & 2x + 3y = 6
\end{align*}
\]

- a) $x = 1.071, y = 1.286$
- b) $x = 1.286, y = 1.071$
- c) $x = 1.078, y = 1.281$
- d) $x = 1.281, y = 1.078$

No, the answer is incorrect.
Score: 0
Accepted Answers:

a) $x = 1.071, y = 1.286$

4) Find the extrema of the function by Lagrange multiplier method

\[
f(x, y) = 2y + x
\]

Subject to $g(x, y) = y^2 + xy - 1 = 0$

- a) $(0, 10) \text{ & } (10, -1)$
- b) $(0, 1) \text{ & } (0, -1)$
- c) $(0, 2) \text{ & } (1, -1)$
- d) $(0, 0) \text{ & } (0, 10)$

No, the answer is incorrect.
Score: 0
Accepted Answers:

b) $(0, 1) \text{ & } (0, -1)$

5) Find the critical points of the function by Lagrange multiplier

\[
\begin{align*}
\text{minimize } & f(x, y) = x^2 + 3xy + y^2 - x + 3y \\
\text{subject to } & g(x, y) = x^2 - y^2 = -1
\end{align*}
\]

- a) $(0, 1) \text{ \& } \left(\frac{3}{4}, -\frac{1}{4}\right)$
- b) $(0, -1) \text{ \& } \left(\frac{3}{4}, \frac{1}{4}\right)$
- c) $(0, 1) \text{ \& } \left(-\frac{3}{4}, -\frac{1}{4}\right)$
- d) $(0, -1) \text{ \& } \left(\frac{3}{4}, -\frac{1}{4}\right)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
Find the optimal solution of the function by Lagrange multiplier method

maximize \( f(x, y) = 20x^{3/2}y \)

subject to \( g(x, y) = x + y = 60 \)

a) (20, 24)

b) (16, 24)

c) (6, 20)

d) (36, 24)

No, the answer is incorrect.
Score: 0
Accepted Answers:
d) (36, 24)
Maximize $50x - 2x^2 + 3y^2 - xy + 95y$

s.t. $x + y = 25$

Then $y = 25 - x$

$$f(x) = 50x - 2x^2 - 3(25 - x)^2 - x(25 - x) + 95(25 - x)$$

$$= 50x - 2x^2 - 3(625 - 50x + x^2) - 25x + x^2 + 2375 - 95x$$

$$= 50x - 2x^2 - 1875 + 150x - 3x^2 - 25x + x^2 + 2375 - 95x$$

$$= 500 + 80x - 4x^2$$

$$f'(x) = 80 - 8x$$

$\Rightarrow f'(x) = 0$ gives $x = 10$

$\therefore y = 15$

Hence $f(x, y) = 50 \times 10 - 2 \times 10^2 - 10 \times 15 - 3 \times 15^2 + 95 \times 15$

$$= 900$$
Form \( 2x + 3y = 6 \), \( y = 2 - \frac{2}{3}x \)

Then \( 4x^2 + 5y^2 \)

\[
= 4x^2 + 5 \left( 2 - \frac{2}{3}x \right)^2 \\
= 4x^2 + 5 \left( 4 - 2 \cdot 2 \cdot \frac{2}{3}x + \frac{4}{9}x^2 \right) \\
= 4x^2 + \frac{20}{9}x^2 - \frac{40}{3}x + 20 \\
= \frac{56}{9}x^2 - \frac{40}{3}x + 20
\]

Then, \( \nabla(4x^2 + 5y^2) = 0 \) gives \( x = 1.0714 \)

Then \( y = 1.286 \)
\[ f(x, y) = 2x + y \\
q(x, y) = y^2 + xy - 1 = 0 \]

The Lagrange function is

\[ L(x, y, \lambda) = 2x + y + \lambda (y^2 + xy - 1) = 0 \]

\[
\frac{\partial L}{\partial x} = 1 + \lambda y = 0 \\
\frac{\partial L}{\partial y} = 2 + \lambda x = 0 \\
\frac{\partial L}{\partial \lambda} = y^2 + xy - 1 = 0
\]

Solving (i), (ii) and (iii) we get the solution

(0, 1) and (0, -1)
\[ f(x,y) = x^2 + 3xy + y^2 - x + 3y \]

Let \( F(x,y,\lambda) = f(x,y) + \lambda g(x,y) \)

\[
\frac{\partial F}{\partial x} = 2x + 3y - 1 + 2\lambda x \tag{1}
\]

\[
\frac{\partial F}{\partial y} = 3x + 2y + 3 + 2\lambda y \tag{2}
\]

\[
\frac{\partial F}{\partial \lambda} = x^2 - y^2 + 1 \tag{3}
\]

We set these equal to zero and multiplying \( \lambda \) by \( y \) and \( x \) to (1) and (2) respectively.

\[
0 = 2xy + 3y^2 - y + 2\lambda xy
\]

\[
0 = 2xy + 3x^2 + 3x + \lambda xy
\]

Subtracting, \( 0 = 3(x^2 - y^2) + 3x - y \)

As, \( 0 = x^2 - y^2 + 1 \), \( y = 1 - 3x \).

By substituting, \( 0 = x^2 - (1 - 3x)^2 + 1 \)

\[
= x^2 - 9x^2 + 6x - 1 + 1
\]

\[
= -8x^2 + 6x
\]

\[
= x(6 - 8x)
\]

So the critical points are \((0,1)\) and \(\left(\frac{3}{4}, -\frac{1}{4}\right)\)
\[ f(x, y) = 20x^{3/2}y \]

\[ g(x, y) = x + y = 60 \]

The Lagrange function is

\[ L(x, y, \lambda) = 20x^{3/2}y + \lambda(x + y - 60) \]

\[ \frac{\partial L}{\partial x} = 30x^{1/2}y + \lambda = 0 \]

\[ \frac{\partial L}{\partial y} = 20x^{3/2} + \lambda = 0 \]

\[ \frac{\partial L}{\partial \lambda} = x + y - 60 = 0 \]

Solving this we get \( x = 36 \)

\( y = 24 \)