1) As \( x_1 \leq x_2 \)
   for monotonically increasing \( h(x) \) and \( f(x_2) \)
   \( f(x_1) \leq f(x_2) \)

2. Self-Explanatory

3. Self-Explanatory

4. 

5. Self-Explanatory

6. \( f(x) = x^3 - 12x \)
   Then, \( f'(x) = 3x^2 - 12 \)
   For getting the critical points \( 3x^2 - 12 = 0 \)
   \( \text{or, } x = \pm \sqrt{4} \)
   Now, \( f''(x) = 6x \)
   \( f''(2) > 0 \) \( f''(-2) < 0 \)
   Hence, the local minima and maxima are at \( x = 2 \) and \( x = -2 \) respectively.
7. \[ f(x) = x^3 - 9x^2 - 48x + 92 \]
   \[ f'(x) = 3x^2 - 18x - 48 \]
   Then, the critical point(s) may be found out by
   \[ f'(x) = 0 \]
   \[ 3x^2 - 18x - 48 = 0 \]
   \[ 2x = -2, 8 \]
   \[ f''(x) = 6x - 18 \]
   Then, \[ f''(-2) = -12 - 18 = -30 < 0 \]
   \[ f''(8) = 30 > 0 \]
   Thus \( x = 8 \) corresponds to a local minimum and \( x = -2 \) corresponds to a local maximum.

8. \[ f(x) = 12x^4 - 32x^3 + 24x^2 - 10 \]
   Then, \[ f'(x) = 48x^3 - 96x^2 + 48x \]
   \[ f''(x) = 144x^2 - 192x + 48 \]
   To get the point of inflexion,
   \[ f''(x) = 0 \Rightarrow 144x^2 - 192x + 48 = 0 \]
   \[ (x - 1)(x - \frac{1}{3}) = 0 \]
   \[ x = 1, \frac{1}{3} \]

<table>
<thead>
<tr>
<th>( -\infty &lt; x &lt; \frac{1}{3} )</th>
<th>( x = \frac{1}{3} )</th>
<th>( \frac{1}{3} &lt; x &lt; 1 )</th>
<th>( x = 1 )</th>
<th>( 1 &lt; x &lt; \infty )</th>
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<td>+</td>
<td>0</td>
<td>-</td>
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Assignment for Non-linear programming problem

week-1

Introduction to NLP

1. For two point \( x_1 \) and \( x_2 \), where \( x_1 \leq x_2 \), \( f(x_1) \) and \( f(x_2) \) would be monotonically increasing if
   a) \( f(x_1) \leq f(x_2) \)
   b) \( f(x_1) < f(x_2) \)
   c) \( f(x_1) > f(x_2) \)
   d) \( f(x_1) \geq f(x_2) \)

2. The function is unimodal in
   a) \([0,1]\)
   b) \([0.5,1]\)
   c) \([1,1.5]\)
   d) \([1,2]\)

3. Solve the optimization problem by graphical method

   \[
   \text{Min} \quad 3x + 2y \\
   \text{s. t.} \quad 9(x-2)^2 + 2x - y \leq 5 \\
   \quad y \leq 5 \\
   \quad x, y \geq 0
   \]

   a) 4.61
   b) 5.90
   c) 3.89
   d) 4.99

4. Find the point of inflection

   \[
   f(x) = 4x^3 - 18x^2 + 27x - 10
   \]

   a) \( x=2 \)
   b) \( x=2.3 \)
   c) \( x=1.7 \)
   d) \( x=1.5 \)
5. Solve the optimization problem by graphical method

\[
\text{Max } 2x + 3y \\
\text{s.t. } x^2 + y^2 \leq 5 \\
x + y \geq 2 \\
x, y \geq 0
\]

a) 16.99  
 b) 10.23  
 c) 8.06  
 d) 13.33

6. Identify local minima and local maxima of \( f(x) = x^3 - 12x \) over the region \(-3 \leq x \leq 3\).

a) \( x = 0, x = 0 \)  
 b) \( x = 2, x = -2 \)  
 c) \( x = 3, x = -3 \)  
 d) \( x = 1, x = -1 \)

7. Find the value of \( x \) for which the local minima in the interval \([1,2]\)

\[
f(x) = \cos(14.5x - 0.3) + x(x + 0.2) + 1.01
\]

a) 1.8  
 b) 1.3  
 c) 1.5  
 d) 1.1

8. Find the point of inflections

\[
f(x) = 12x^4 - 32x^3 + 24x^2 - 10
\]

a) 1 and 0.33  
 b) 2 and 1.33  
 c) -1 and -1.33  
 d) 0 and -0.33
Answer

1. a,
2. c,
3. a
4. d
5. c
6. b
7. d
8. a