ASSIGNMENT 4

Q.1. If the dual problem has unbounded solution, then what can we say about the solution of the primal?

(a) Feasible solution          (b) No feasible solution
(c) Unbounded solution         (d) Nothing can be said

Q.2. How many non-negative variables will be there in the dual of the following problem?

Minimize \( z = 3x_1 - 2x_2 + 4x_3 \)

subject to

\[
\begin{align*}
3x_1 + 5x_2 + 4x_3 & \geq 7 \\
6x_1 + x_2 + 3x_3 & \geq 4 \\
7x_1 - 2x_2 - x_3 & \leq 10 \\
x_1 - 2x_2 + 5x_3 & \geq 3 \\
4x_1 + 7x_2 - 2x_3 & \geq 2 \\
x_1, x_2, x_3 & \geq 0.
\end{align*}
\]

(a) 3          (b) 4          (c) 5          (d) None of the above

Q.3. How many constraints (excluding non-negativity constraints) will be there in the dual of the following problem?

Maximize \( z = 2x_1 - 6x_2 \)

subject to

\[
\begin{align*}
x_1 - 3x_2 & \leq 6 \\
2x_1 + 4x_2 & \geq 8 \\
x_1 - 3x_2 & \geq -6 \\
x_1, x_2 & \geq 0.
\end{align*}
\]

(a) 1          (b) 2          (c) 3          (d) None of the above

Q.4. Construct the constraints of the dual of the following linear programming problem:

Maximize \( z = 2x_1 + 3x_2 + x_3 \)

subject to

\[
\begin{align*}
4x_1 + 3x_2 + x_3 & = 6 \\
x_1 + 2x_2 + 5x_3 & = 4 \\
x_1, x_2, x_3 & \geq 0.
\end{align*}
\]
(a) $4u + v \geq 2$, $3u + 2v \geq 3$, $u + 5v \geq 1$, $u, v \geq 0$

(b) $4u + v \geq 2$, $3u + 2v \geq 3$, $u + 5v \geq 1$, $u, v$ are unrestricted in sign

(c) $4u + v \leq 2$, $3u + 2v \leq 3$, $u + 5v \leq 1$, $u, v$ are unrestricted in sign

(d) None of the above

Q.5. Give the dual of the following linear programming problem:

Maximize $z = 3x_1 - 2x_2$

subject to

$x_1 \leq 4$

$x_2 \leq 6$

$x_1 + x_2 \leq 5$

$-x_2 \leq -1$

$x_1, x_2 \geq 0$

(a) Min $w = 4v_1 + 6v_2 + 5v_3 - v_4$, s.t. $v_1 + v_3 \geq 3$, $v_4 - v_2 - v_3 \leq 2$, $v_1, v_2, v_3, v_4 \geq 0$

(b) Max $w = -4v_1 - 6v_2 - 5v_3 + v_4$, s.t. $v_1 + v_3 \leq 3$, $v_4 - v_2 - v_3 \geq 2$, $v_1, v_2, v_3, v_4 \geq 0$

(c) Min $w = 4v_1 + 6v_2 + 5v_3 - v_4$, s.t. $v_1 + v_3 \geq 3$, $v_4 - v_2 - v_3 \geq 2$, $v_1, v_2, v_3, v_4 \geq 0$

(d) None of the above

Q.6. Construct the constraints of the dual for the following primal problem:

Minimize $z = x_1 - 2x_2 + x_3$

subject to

$3x_1 - x_2 + 5x_3 \leq 17$

$x_1 + 2x_2 - x_3 = 25$

$2x_1 - x_2 + 5x_3 \geq 57$

$x_1, x_2 \geq 0$, $x_3$ is unrestricted in sign.

(a) $-3v_1 + v_2 + 2v_3 \geq 1$, $v_1 + 2v_2 - v_3 \geq -2$, $-5v_1 - v_2 + 5v_3 \geq 1$, $v_1, v_2 \geq 0$, $v_3$ is unrestricted in sign

(b) $-3v_1 + v_2 + 2v_3 \leq 1$, $v_1 + 2v_2 - v_3 \leq -2$, $-5v_1 - v_2 + 5v_3 \leq 1$, $v_2, v_3 \geq 0$, $v_1$ is unrestricted in sign

(c) $-3v_1 + v_2 + 2v_3 \leq 1$, $v_1 + 2v_2 - v_3 \leq -2$, $-5v_1 - v_2 + 5v_3 = 1$, $v_1, v_3 \geq 0$, $v_2$ is unrestricted in sign

(d) None of the above
Q.7. Find the optimal solution of the dual of the given problem :

Maximize \( z = 2x_1 + 3x_2 \)

subject to

\(-x_1 + 2x_2 \leq 4\)
\(x_1 + x_2 \leq 6\)
\(x_1 + 3x_2 \leq 9\)
\(x_1, x_2 \geq 0.\)

(a) \( w_{min} = \frac{27}{2} \)  (b) \( w_{min} = -\frac{27}{2} \)  (c) No feasible solution  (d) Unbounded solution

Q.8. Find the optimal solution of the dual of the given problem :

Minimize \( z = x_1 - x_2 \)

subject to

\(2x_1 + x_2 \geq 2\)
\(-x_1 - x_2 \geq 1\)
\(x_1, x_2 \geq 0.\)

(a) \( w_{max} = 120 \)  (b) \( w_{max} = 75 \)  (c) No feasible solution  (d) Unbounded solution
ANSWERS

Q.1.  (b)
Q.2.  (c)
Q.3.  (b)
Q.4.  (b)
Q.5.  (a)
Q.6.  (c)
Q.7.  (a)
Q.8.  (d)
Solutions to Problems of ASSIGNMENT - 4

2. The given LPP is a minimization problem.
So we arrange all the constraints to "≥" type —

\[ 3u_1 + 5 u_2 + 4 u_3 \geq 7 \]
\[ 6 u_1 + u_2 + 3 u_3 \geq 4 \]
\[ -7 u_1 + 2 u_2 + u_3 \geq -10 \]
\[ u_1 - 2 u_2 + 5 u_3 \geq 3 \]
\[ 4 u_1 + 7 u_2 - 2 u_3 \geq 2 \]
\[ u_1, u_2, u_3 \geq 0 \]

: The required dual to the given problem is

\[ \text{Maximize } W = 7 u_1 + 4 u_2 - 10 u_3 + 3 u_4 + 2 u_5 \]

subject to

\[ 3 u_1 + 6 u_2 - 7 u_3 + u_4 + 4 u_5 \leq 3 \]
\[ 5 u_1 + u_2 + 2 u_3 - 2 u_4 + 7 u_5 \leq -2 \]
\[ 4 u_1 + 3 u_2 + u_3 + 5 u_4 - 2 u_5 \leq 4 \]
\[ u_j \geq 0 \quad (j = 1, 2, \ldots, 5) \]

3. The given LPP is a maximization problem.
So the constraints are arranged as "≤" type —
\[ n_1 - 3n_2 \leq 6 \]
\[ -2n_1 - 4n_2 \leq -8 \]
\[ -n_1 + 3n_2 \leq 6 \]
\[ n_1, n_2 \geq 0 \]

\text{: The required dual problem is given by —}

\text{Minimize } W = 6v_1 - 8v_2 + 6v_3 \\
\text{s.t. } v_1 - 2v_2 - v_3 \geq 2 \\
\quad -3v_1 - 4v_2 + 3v_3 \geq -6 \\
\quad v_1, v_2, v_3 \geq 0.

4. The given LPP is a maximization problem.

The constraints can be rewritten as —

\[ 4n_1 + 3n_2 + n_3 \leq 6 \]
\[ 4n_1 + 3n_2 + n_3 \geq 6 \]
\[ n_1 + 2n_2 + 5n_3 \leq 4 \]
\[ n_1 + 2n_2 + 5n_3 \geq 4 \]
\[ n_1, n_2, n_3 \geq 0 \]

\[ \Rightarrow \]

\[ -4n_1 - 3n_2 - n_3 \leq -6 \]
\[ n_1 + 2n_2 + 5n_3 \leq 4 \]
\[ -n_1 - 2n_2 - 5n_3 \leq -4 \]
\[ n_1, n_2, n_3 \geq 0 \]

\text{: The required dual is}

\text{Minimize } W = 6v_1 - 6v_2 + 4v_3 - 4v_4 \\
\text{s.t. } 4v_1 - 4v_2 + v_3 - v_4 \geq 2 \\
\quad 3v_1 - 3v_2 + 2v_3 - 2v_4 \geq 3 \\
\quad v_1 - v_2 + 5v_3 - 5v_4 \geq 1 \\
\quad v_1, v_2, v_3, v_4 \geq 0.

\text{Let, } u = v_1 - v_2 \text{ and } v = v_3 - v_4 \text{ when } u \text{ and } v \text{ are both unrestricted in sign (non-negative, non-positive or 0).}
The dual problem becomes

\[ \text{Min} \quad W = 6u + 4v \]
\[ \text{s.t.} \quad 4u + v \geq 2 \]
\[ 3u + 2v \geq 3 \]
\[ u + 5v \geq 1 \]
\[ u, v \text{ are unrestricted in sign.} \]

5. The given problem is of minimization type and all the constraints are of \( \leq \) type.

The required dual is given by

\[ \text{Minimize} \quad W = 4u_1 + 6u_2 + 5u_3 - u_4 \]
\[ \text{s.t.} \quad u_1 + u_3 \geq 3 \]
\[ u_2 + u_3 - u_4 \geq -2 \]
\[ u_1, u_2, u_3, u_4 \geq 0 \]

6. The given problem is of minimization type.
The constraints can be restated as

\[ -3u_1 + u_2 - 5u_3 \geq -17 \]
\[ u_1 + 2u_2 - u_3 \geq 25 \]
\[ -u_1 - 2u_2 + u_3 \geq -25 \]
\[ 2u_1 - u_2 + 5u_3 \geq 57 \]
\[ u_1, u_2 \geq 0 \text{ and } u_3 \text{ is unrestricted in sign.} \]

So, we take \( u_3 = u_3' - u_3'' \) when \( u_3', u_3'' \geq 0 \).
The given problem can be rewritten as:

\[
\begin{align*}
\text{Min } Z &= \eta_1 - 2\eta_2 + \eta_3'' - \eta_3'''
\end{align*}
\]

s.t.

\[
\begin{align*}
-3\eta_1 + \eta_2 - 5\eta_3' + 5\eta_3'' &\geq -17 \\
\eta_1 + 2\eta_2 - \eta_3' + \eta_3''' &\geq 25 \\
-\eta_1 - 2\eta_2 + \eta_3' - \eta_3''' &\geq -25 \\
2\eta_1 - \eta_2 + 5\eta_3' - 5\eta_3'' &\geq 57 \\
\eta_1, \eta_2, \eta_3', \eta_3'' &\geq 0.
\end{align*}
\]

The required dual of the given problem is:

\[
\begin{align*}
\text{Maximize } W &= -17\eta_1 + 25\eta_2 - 25\eta_2'' + 57\eta_3 \\
s.t.
-3\eta_1 + \eta_2' - \eta_2'' + 2\eta_3 &\leq 1 \\
\eta_1 + 2\eta_2' - 2\eta_2'' - \eta_3 &\leq -2 \\
-5\eta_1 - \eta_2' + \eta_2'' + 5\eta_3 &\leq 1 \\
5\eta_1 + \eta_2' - \eta_2'' - 5\eta_3 &\leq -1 \\
\eta_1, \eta_2', \eta_2'', \eta_3 &\geq 0.
\end{align*}
\]

We take \(\eta_2' - \eta_2'' = \eta_2\) when \(\eta_2\) is unrestricted in sign.

The dual problem becomes:

\[
\begin{align*}
\text{Max } W &= -17\eta_1 + 25\eta_2 + 57\eta_3 \\
s.t.
-3\eta_1 + \eta_2 + 2\eta_3 &\leq 1 \\
\eta_1 + 2\eta_2 - \eta_3 &\leq -2 \\
-5\eta_1 - \eta_2 + 5\eta_3 &\leq 1 \\
5\eta_1 + \eta_2 - 5\eta_3 &\leq -1 \\
\eta_1, \eta_3 &\geq 0 & \text{and } \eta_2 \text{ is unrestricted in sign.}
\end{align*}
\]
The last 2 constraints can be together written as

\[-5u_1 - u_2 + 5u_3 = 1.\]

The dual problem is

\[
\begin{align*}
\text{Max} & \quad W = -17v_1 + 25v_2 + 57v_3 \\
\text{s.t.} & \quad -3v_1 + v_2 + 2v_3 \leq 1 \\
& \quad v_1 + 2v_2 - v_3 \leq -2 \\
& \quad -5v_1 - v_2 + 5v_3 = 1 \\
& \quad v_1, v_3 \geq 0 \quad \text{and} \quad v_2 \text{ is unrestricted in sign}
\end{align*}
\]

6. We construct the simplex table:

\[
\begin{align*}
\text{Max} \quad z = & \quad n_1 + 4n_2 - 2n_3 + 3n_4 + n_5 + 0n_6 + 0n_7 + 0n_8 \\
\text{s.t.} \quad & \quad n_1 - 3n_2 + n_3 + 2n_4 + 6n_5 + n_6 = 3 \\
& \quad 2n_1 + n_2 + 3n_4 + 2n_5 + n_7 = 6 \\
& \quad 4n_1 + n_2 - n_4 + n_5 + n_8 = 2 \\
& \quad n_j \geq 0 \quad (j = 1, 2, \ldots, 8)
\end{align*}
\]

\[
\begin{array}{cccccccccccc}
\text{B} & \text{C} & \chi_0 & b & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\
\hline
a_6 & 0 & n_6 & 3 & 1 & -3 & 1 & 2 & 6 & 1 & 0 & 0 \\
a_7 & 0 & n_7 & 6 & 2 & 1 & 0 & 3 & 2 & 0 & 1 & 0 \\
a_8 & 0 & n_8 & 2 & 4 & 1 & 1 & -1 & 1 & 0 & 0 & 1 \\
\hline
\text{z} - c & -1 & -4 & 2 & -3 & +1 & 0 & 0 & 0 \\
\hline
a_6 & 0 & n_6 & 9 & 13 & 0 & 1 & -1 & 9 & 1 & 0 & 3 \\
a_7 & 0 & n_7 & 4 & -2 & 0 & 0 & -4 & 1 & 0 & 1 & 1 \\
a_8 & 4 & n_2 & 2 & 4 & 1 & 0 & -1 & 1 & 0 & 0 & 1 \\
\hline
\text{z} - c & 15 & 0 & 2 & -1 & 5 & 0 & 0 & 0 & 4
\end{array}
\]
7. Dual of the given problem is

Minimize \( W = 4u_1 + 6u_2 + 9u_3 \)

S.t.
- \( u_1 + u_2 + u_3 \geq 2 \)
- \( 2u_1 + u_2 + 3u_3 \geq 3 \)
- \( u_1, u_2, u_3 \geq 0 \).

We now use the simplex method to solve the dual.

Maximize \( W = -4v_1 - 6v_2 - 9v_3 + 0v_4 + 0v_5 - Mv_6 - Mv_7 \)

S.t.
- \( u_1 + u_2 + u_3 - v_4 + v_6 = 2 \)
- \( 2u_1 + u_2 + 3u_3 - v_5 + v_7 = 3 \)
- \( v_j \geq 0 \) \((j = 1, 2, \ldots, 7)\).

\( \text{C} \)

\begin{array}{ccccccccc}
\text{C} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & \text{Min} \\
\text{b} & b & b & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & \text{Max} \\
\text{M} & a_1 & v_1 & a_4 & a_7 & a_6 & a_6 & a_7 & v_7 & v_7 & a_7 \\
\text{M} & a_7 & v_7 & 3 & 2 & 1 & 3 & 0 & -1 & 0 & 1 & 1 \\
\text{M} & v_1 & v_2 & 1 & -\sqrt{3} & \sqrt{3} & 0 & -1 & 1/3 & 1 & -1/3 & 1 \\
\text{M} & v_3 & 1 & 2/3 & 1/3 & 1/3 & 0 & -1/3 & 0 & 1/3 & 0 & 1/3 \\
\end{array}

\( \text{Max} \)

\( \text{C} \)

\begin{array}{ccccccccc}
\text{M} & 3/2 & -2/3 & 0 & M & -M/3 & 0 & 4M/3 & -3 \\
\text{M} & 2 & -2/3 & 0 & M & -M/3 & 0 & 4M/3 & -3 \\
\text{M} & -2 & 3/2 & 0 & M & -M/3 & 0 & 4M/3 & -3 \\
\end{array}

\( \text{C} \)

\begin{array}{ccccccccc}
\text{M} & 1 & 0 & 0 & 1 & 1/2 & 1/2 & 1/2 & 1/2 \\
\text{M} & 1 & 0 & 0 & 1 & 1/2 & 1/2 & 1/2 & 1/2 \\
\text{M} & 1 & 0 & 0 & 1 & 1/2 & 1/2 & 1/2 & 1/2 \\
\end{array}
\[ z_j - g_j \geq 0 \quad \forall j \]

The optimal solution for the dual problem is:

\[ u_1 = 0, \quad u_2 = \frac{3}{2}, \quad u_3 = \frac{1}{2} \]

\[ W_{\text{max}} = -W_{\text{min}} = -\frac{27}{2} = W_{\text{min}} = \frac{27}{2} \]

The artificial variables are \( u_6 \) and \( u_7 \).

The optimal solution is obtained from the index row after deleting the penalty cost \( M \) and changing sign of the net evaluations corresponding to \( a_6 \) and \( a_7 \).

Thus the optimal solution is:

\[ u_1 = \frac{9}{2}, \quad u_2 = \frac{3}{2} \]

\[ z_{\text{max}} = \frac{27}{2} \]

8. The dual of the given problem is:

Maximize \( W = 2u_1 + u_2 \)

s.t. \[ 2u_1 - u_2 \leq 1 \]
\[ u_1 - u_2 \leq -1 \]
\[ u_1, u_2 \geq 0 \]

Introducing slack, surplus & artificial variables, we have:

Minimize \[ W = 2u_1 + u_2 + 0. u_3 + 0. u_4 - M u_5 \]

s.t. \[ 2u_1 - u_1 + u_3 = 1 \]
\[ -u_1 + u_2 - u_4 + u_5 = 1 \]
\[ u_j \geq 0 \quad \forall j = 1, 2, \ldots, 5. \]
\[
\begin{align*}
\begin{array}{ccccccc}
 & c_1 & c_2 & c_3 & c_4 & c_5 & \text{Min} \\
\text{cB} & 0 & 0 & 0 & 0 & -M & -M \\
0 & a_1 & a_2 & a_3 & a_4 & a_5 & 0 \\
-M & v_1 & v_2 & v_3 & v_4 & v_5 & 0 \\
z_j-c_j & M_2 & M_3 & M_4 & M_5 & M_6 & 0 \\
0 & a_3 & a_4 & a_5 & a_6 & a_7 & 0 \\
1 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & 1 \\
z_j-c_j & -3 & 0 & 0 & 0 & 0 & 1 \\
2 & a_1 & a_2 & a_3 & a_4 & a_5 & 0 \\
1 & a_6 & a_7 & a_8 & a_9 & a_{10} & 0 \\
z_j-c_j & 0 & 0 & 0 & 0 & 0 & 2 \\
\end{array}
\end{align*}
\]

\[z_4-c_4 = -4 < 0 \text{ but } y_{14} \text{ and } y_{24} \text{ are both -ve}\]

\[\therefore \text{The dual problem has unbounded solution}\]

\[\therefore \text{We can conclude that the primal problem will have no feasible solution.}\]