ASSIGNMENT 1
(Based on BFS, Formulation of L.P.P. and Graphical Method)

Q.1. Two linearly independent equations with four variables are given below:

\[ 4x_1 + 2x_2 + 3x_3 - 8x_4 = 6 \]
\[ 3x_1 + 5x_2 + 4x_3 - 6x_4 = 8 \]

How many basic solutions are there?
(a) 2  (b) 3  (c) 4  (d) 5

Q.2.

\[ 2x_1 + 3x_2 - x_3 + 4x_4 = 8 \]
\[ x_1 - 2x_2 + 6x_3 - 7x_4 = -3 \]
\[ x_1, x_2, x_3, x_4 \geq 0 \]

Write down a basic feasible and a basic non-feasible solution of the above set of equations.

(a) (0, 0, 2, 1) and \( \left( \frac{14}{13}, 0, \frac{45}{13} \right) \)
(b) (1, 2, 0, 0) and \( \left( -\frac{14}{13}, 0, \frac{45}{13} \right) \)
(c) (0, 0, 2, 1) and \( \left( 0, \frac{45}{13}, 0, -\frac{14}{13} \right) \)
(d) (1, 2, 0, 0) and \( \left( \frac{45}{13}, 0, -\frac{14}{13} \right) \)

Q.3. The two linearly independent equations with three variables are given below:

\[ 2x_1 - 3x_2 + 5x_3 = 10 \]
\[ 4x_1 + x_2 + 10x_3 = 20 \]

Find, if possible, a basic solution with \( x_2 \), a non-basic variable.

(a) \( \left[ \frac{5}{2}, 0, 1 \right] \)  (b) \( [-5, 0, 4] \)  (c) \( \left[ \frac{7}{4}, 0, \frac{13}{10} \right] \)  (d) Such a solution does not exist

Q.4. A factory is engaged in manufacturing three products A, B and C which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively. Similarly they are 3, 1 and 3 hours for one unit of B and 1, 3 and 1 hours for one unit of C. the profits on A, B and C are Rs. 2, Rs. 2 and Rs. 4 per unit respectively. Assuming that there are available 300 hours of lathe time, 300 hours of grinder time and 240 hours of assembly time, which of the following represents the set of constraints for the equivalent L.P.P. formulated from the above data?

(a) \( 2x_1 + x_2 + x_3 \leq 300 \), \( 3x_1 + x_2 + 3x_3 \leq 300 \), \( x_1 + 3x_2 + x_3 \leq 240 \), \( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \)
(b) \( 2x_1 + 3x_2 + x_3 \leq 300 \), \( x_1 + x_2 + 3x_3 \leq 300 \), \( x_1 + 3x_2 + x_3 \leq 240 \), \( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \)
(c) \( 2x_1 + x_2 + x_3 \geq 300 \), \( 3x_1 + x_2 + 3x_3 \geq 300 \), \( x_1 + 3x_2 + x_3 \geq 240 \), \( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \)

(d) \( 2x_1 + x_2 + x_3 \leq 300 \), \( 3x_1 + x_2 + 3x_3 \leq 300 \), \( x_1 + 3x_2 + x_3 \leq 240 \), \( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \)
Q.5. Suppose in a hospital, it is decided that each patient should be given at least \( N_1, N_2, N_3, N_4 \) quantities of 4 nutrients daily. There are 5 such foods available which contain these nutrients in different proportions. Suppose \( a_{ij} \) \((i=1,2,3,4; j=1,2,3,4,5)\) is the quantity of the \( i^{th} \) nutrient in one unit of the \( j^{th} \) food. The price per unit of the \( j^{th} \) food is \( c_j \). The problem is how to find a programme for the best diet, that is, the combination of foods that can be supplied at minimum cost satisfying the daily requirements of the nutrients by the patients.

Write down the mathematical formulation of the above mentioned problem as L.P.P.

(a) \( \text{Min } z = \sum_{j=1}^{4} c_j x_j \text{ s.t. } \sum_{j=1}^{4} a_{ij} x_j \geq N_i \text{ (for } i=1, 2, 3, 4, 5, \text{)} \), \( x_j \geq 0 \)
(b) \( \text{Min } z = \sum_{j=1}^{5} c_j x_j \text{ s.t. } \sum_{j=1}^{5} a_{ij} x_j \leq N_i \text{ (for } i=1, 2, 3, 4 \text{)} \), \( x_j \geq 0 \)
(c) \( \text{Min } z = \sum_{j=1}^{5} c_j x_j \text{ s.t. } \sum_{j=1}^{5} a_{ij} x_j \geq N_i \text{ (for } i=1, 2, 3, 4 \text{)} \), \( x_j \geq 0 \)
(d) \( \text{Min } z = \sum_{j=1}^{4} c_j x_j \text{ s.t. } \sum_{j=1}^{4} a_{ij} x_j \leq N_i \text{ (for } i=1, 2, 3, 4, 5, \text{)} \), \( x_j \geq 0 \)

Q.6. Solve the following L.P.P.

\[
\text{Max } z = 5x_1 - 2x_2 \\
\text{subject to } 5x_1 + 6x_2 \geq 30 \\
9x_1 - 2x_2 = 72 \\
x_2 \leq 9 \\
x_1, x_2 \geq 0
\]

and choose the correct option denoting the solution to the problem.

(a) Unbounded solution \hspace{1cm} (b) \( x_1 = 8, x_2 = 0 \) \hspace{1cm} (c) \( x_1 = 10, x_2 = 9 \) \hspace{1cm} (d) No feasible solution

Q.7. The optimal value of the objective function for the following L.P.P.

\[
\text{Max } z = 4x_1 + 3x_2 \\
\text{subject to } x_1 + x_2 \leq 50 \\
x_1 + 2x_2 \leq 80 \\
2x_1 + x_2 \geq 20 \\
x_1, x_2 \geq 0
\]

is \((\text{a) } 200 \) \hspace{1cm} (\text{b) } 330 \) \hspace{1cm} (\text{c) } 420 \) \hspace{1cm} (\text{d) } 500 \)
Q.8. How many basic solutions are there in the following linearly independent set of equations? Find all of them.

\[ \begin{align*}
2x_1 - x_2 + 3x_3 + x_4 &= 6 \\
4x_1 - 2x_2 - x_3 + 2x_4 &= 10.
\end{align*} \]

(a) 2 basic solutions  (b) 3 basic solutions  (c) 4 basic solutions  (d) 5 basic solutions

Q.9. Solve graphically:

Minimize \( z = x_1 + x_2 \)

subject to \( \begin{align*}
5x_1 + 9x_2 &\leq 45 \\
x_1 + x_2 &\geq 2 \\
x_2 &\leq 4 \\
x_1, x_2 &\geq 0.
\end{align*} \)

(a) \( z_{\text{min}} = 2 \)  (b) \( z_{\text{min}} = 4 \)  (c) \( z_{\text{min}} = 9 \)  (d) \( z_{\text{min}} = \frac{29}{5} \)

Q.10. The manager of an oil refinery must decide on the optimal mix of two possible blending processes of which the inputs and outputs per production run are as follows:

<table>
<thead>
<tr>
<th>Process</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crude A</td>
<td>Crude B</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

The maximum amounts available of crudes A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 300 and Rs. 400 respectively. Formulate an L.P.P. for maximization of the profit and solve it graphically.

(a) Optimal solution is \( \left( \frac{400}{13}, \frac{150}{13} \right) \)  (b) Optimal solution is \( (40, 0) \)

(c) Unbounded solution  (d) No feasible solution

Q.11.

Maximize \( z = 3x_1 + 4x_2 \)
subject to  

\begin{align*}
    & x_1 - x_2 \leq -1 \\
    & -x_1 + x_2 \leq 0 \\
    & x_1, x_2 \geq 0.
\end{align*}

(a) No feasible solution  
(b) Unbounded solution

Q.12. An agricultural farm has 180 tons of Nitrogen fertilizers, 250 tons of Phosphate and 220 tons of Potash. It is able to sell 3:3:4 mixtures of these substances at a profit of Rs. 15 per ton and 2:4:2 mixture at a profit of Rs. 12 per ton respectively. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. (Let \( x_1 \) and \( x_2 \) tons of these two mixtures be produced)

(a) \( \text{Max } z = 15x_1 + 12x_2 \)  s.t.  \( 3x_1 + 2x_2 \leq 180, 3x_1 + 4x_2 \leq 250, 4x_1 + 2x_2 \leq 220, \ x_1, x_2 \geq 0 \)
(b) \( \text{Max } z = 15x_1 + 12x_2 \)  s.t.  \( 5x_1 + 6x_2 \leq 3600, 5x_1 + 3x_2 \leq 2500, 5x_1 + 8x_2 \leq 4400, \ x_1, x_2 \geq 0 \)
(c) \( \text{Max } z = 15x_1 + 12x_2 \)  s.t.  \( 6x_1 + 5x_2 \leq 3600, 3x_1 + 5x_2 \leq 2500, 8x_1 + 5x_2 \leq 4400, \ x_1, x_2 \geq 0 \)
(d) \( \text{Max } z = 15x_1 + 12x_2 \)  s.t.  \( 2x_1 + 3x_2 \leq 180, 4x_1 + 3x_2 \leq 250, 2x_1 + 4x_2 \leq 220, \ x_1, x_2 \geq 0 \)

Q.13. Solve graphically the L.P.P.

\begin{align*}
\text{Minimize } z &= -2x_1 + x_2 \\
\text{subject to } & x_1 + x_2 \geq 6 \\
& 3x_1 + 2x_2 \geq 16 \\
& x_2 \leq 9 \\
& x_1, x_2 \geq 0.
\end{align*}

(a) \((x_1, x_2) = (6, 0)\)  
(b) \((x_1, x_2) = (0, 10)\)  
(c) No feasible solution 
(d) Unbounded solution
ANSWERS

Q.1.  (d)
Q.2.  (d)
Q.3.  (d)
Q.4.  (b)
Q.5.  (c)
Q.6.  (b)
Q.7.  (a)
Q.8.  (b)
Q.9.  (a)
Q.10. (a)
Q.11. (a)
Q.12. (c)
Q.13. (d)
Solution to problems of ASSIGNMENT - 1

1. Here the equations can be written as

\[ a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = b \]

where

\[ a_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \quad a_3 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad a_4 = \begin{pmatrix} -8 \\ -6 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \]

From the 4 vectors \( a_1, a_2, a_3 \) and \( a_4 \),

taking 2 at a time, \( 4 \times 2 = 6 \) square sub-matrices are

\[ B_1 = (a_1, a_2) = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}, \quad B_2 = (a_1, a_3) = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}, \]

\[ B_3 = (a_1, a_4) = \begin{pmatrix} 4 & -8 \\ 3 & -6 \end{pmatrix}, \quad B_4 = (a_2, a_3) = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}, \]

\[ B_5 = (a_2, a_4) = \begin{pmatrix} 2 & -8 \\ 5 & -6 \end{pmatrix}, \quad B_6 = (a_3, a_4) = \begin{pmatrix} 3 & -8 \\ 4 & -6 \end{pmatrix}. \]

\[ \det B_1 = 14 \neq 0, \quad \det B_2 = 7 \neq 0, \quad \det B_3 = 0, \]

\[ \det B_4 = -7 \neq 0, \quad \det B_5 = 28 \neq 0, \quad \det B_6 = 14 \neq 0 \]

Here, 5 square sub-matrices are non-singular,

\[ \exists 5 \text{ basic matrices and corresponding to which } \]

\[ \exists 5 \text{ basic solutions.} \]

Ans: There are 5 basic solutions.
2. We write the system of equations as

\[ a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 = b \]

where, \( a_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \), \( a_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \), \( a_3 = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \),

\( a_4 = \begin{pmatrix} 4 \\ -7 \end{pmatrix} \), \( b = \begin{pmatrix} 8 \\ -3 \end{pmatrix} \).

There are 4 sub-matrices as:

\[ B_1 = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad B_2 = \begin{pmatrix} a_1 \\ a_3 \end{pmatrix}, \quad B_3 = \begin{pmatrix} a_1 \\ a_4 \end{pmatrix}, \quad B_4 = \begin{pmatrix} a_2 \\ a_3 \end{pmatrix}, \quad B_5 = \begin{pmatrix} a_2 \\ a_4 \end{pmatrix}, \quad B_6 = \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} \]

\[ \det B_1 = -7 \neq 0, \quad \det B_2 \neq 0, \quad \det B_i \neq 0 \quad (i = 3, 4, 5, 6) \]

Now, \( x_{B_i} = B_i^{-1} b \quad (i = 1, 2, \ldots, 6) \)

will give the vectors of the basic variables associated to each \( B_i \).

\[ \begin{align*}
\mathbf{x}_{B_1} & = B_1^{-1} b = -\frac{1}{7} \begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 8 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\
\mathbf{x}_{B_2} & = B_2^{-1} b = \frac{1}{13} \begin{pmatrix} 6 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 8 \\ -3 \end{pmatrix} = \begin{pmatrix} 45/13 \\ -14/13 \end{pmatrix}
\end{align*} \]

So we obtain one basic feasible solution as

\[ (x_1, x_2, x_3, x_4) = (1, 2, 0, 0) \]

and one basic non-feasible solution as

\[ (x_1, x_2, x_3, x_4) = \left(\frac{45}{13}, 0, -\frac{14}{13}, 0\right) \]
3. The given equations are linearly independent.

But the vectors

\[ a_1 = \left( \frac{2}{4} \right) \quad \text{and} \quad a_3 = \left( \frac{5}{10} \right) \]

are not since

\[ B = (a_1, a_3) = \left( \begin{array}{c} 2 \\ 4 \\ 5 \\ 10 \end{array} \right) \quad \text{and} \quad \det B = 0. \]

\[ \Rightarrow \] If we initially assume \( x_2 \) as a non-basic variable whose value is 0, the given equations reduce to

\[
\begin{align*}
2x_1 + 5x_3 &= 10 \\
4x_1 + 10x_3 &= 20
\end{align*}
\]

which represent the same equation, having an infinite number of solutions, but none is basic.

\[ \therefore \] For the given set of equations, a basic solution with \( x_2 \), a non-basic variable, does not exist.

4. The given data, when formulated mathematically, represents the following LPP:

\[
\begin{align*}
\text{Max} \quad z &= 2x_1 + 2x_2 + 4x_3 \\
\text{S. to} \quad 2x_1 + 3x_2 + x_3 &\leq 300 \\
&\quad x_1 + x_2 + 3x_3 \leq 300 \\
&\quad x_1 + 3x_2 + x_3 \leq 240 \\
&\quad x_1, x_2, x_3 \geq 0
\end{align*}
\]

\[ \therefore \] Option (b) is correct.
5. Suppose \( x_1, x_2, x_3, x_4, x_5 \) be the quantities of 5 foods respectively which constitute the daily diet of each patient. Then the problem which will is to find \( x_1, x_2, x_3, x_4, x_5 \) minimize the total cost

\[
Z = c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 + c_5 x_5
\]

Subject to

\[
\begin{align*}
    a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 + a_{15} x_5 & \geq N_1 \\
    a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 + a_{25} x_5 & \geq N_2 \\
    a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 + a_{35} x_5 & \geq N_3 \\
    a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 + a_{45} x_5 & \geq N_4
\end{align*}
\]

\( x_1, x_2, x_3, x_4, x_5 \geq 0 \)

6. Graphical Solution —

\( \overrightarrow{AD}, \overrightarrow{CD} \) and \( \overrightarrow{DE} \) denote the straight lines given by the equations \( 5x_1 + 6x_2 = 30 \), \( x_2 = 9 \) and \( 9x_1 - 2x_2 = 72 \) respectively.
The feasible region satisfied by all the constraints is the segment DE of the line DE.

The extreme points of the convex region are D (10, 9) and E (8, 0).

Now, 
\[ Z_1 = 5 \times 10 - 2 \times 9 = 32 \] for \( x_1 = 10, x_2 = 9 \) at D
\[ Z_2 = 5 \times 8 - 2 \times 0 = 40 \] for \( x_1 = 8, x_2 = 0 \) at E.

\[ \therefore \text{Max} \ Z = Z_2 = 40 \] for \( x_1 = 8, x_2 = 0 \) at E.

\[ \therefore \text{Option (b)} \] is the correct option.

The feasible region satisfied by the given constraints is shown below.

The corner points of the feasible region, as seen from the graph, are \((10, 0), (50, 0), (20, 30), (0, 40), (0, 20)\) and the values of the objective function at these points are respectively
40, 200, 170, 120 and 60.

\[ \therefore Z_{\text{max}} = 200 \] at \((50, 0)\)

Ans. Option (a)
8. Here,
\[ a_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \quad a_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \quad a_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \quad a_4 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \]
and \[ b = \begin{pmatrix} 6 \\ 10 \end{pmatrix} \]

Thus, we can be at most \( 4C_2 = 6 \) basic solutions.

The 6 square sub-matrices taking two at a time from \( a_1, a_2, a_3, a_4 \) are:

\[ B_1 = (a_1, a_2) = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix}, \quad B_2 = (a_1, a_3) = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} \]

\[ B_3 = (a_1, a_4) = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}, \quad B_4 = (a_2, a_3) = \begin{pmatrix} -1 & 3 \\ -2 & -1 \end{pmatrix} \]

\[ B_5 = (a_2, a_4) = \begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}, \quad B_6 = (a_3, a_4) = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \]

\[ \therefore \text{det} \ B_1 = 0, \quad \text{det} \ B_2 = -14 \neq 0, \quad \text{det} \ B_3 = 0, \]
\[ \text{det} \ B_4 = 7 \neq 0, \quad \text{det} \ B_5 = 0, \quad \text{det} \ B_6 = 7 \neq 0 \]

\( \Rightarrow \) only 3 non-singular matrices.

\( \Rightarrow \) only 3 basic solutions.

\( \therefore \) The basic solutions are obtained as follows:

\[ \chi_{B_2} = B_2^{-1} b = \frac{1}{14} \begin{pmatrix} -1 & -3 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 18/7 \\ -2/7 \end{pmatrix} \]

\( \Rightarrow \) Basic solution = \( \left( \frac{18}{7}, 0, \frac{2}{7}, 0 \right) \).

\[ \chi_{B_4} = B_4^{-1} b = \frac{1}{7} \begin{pmatrix} -1 & -3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} -36/7 \\ -2/7 \end{pmatrix} \]

\( \Rightarrow \) Basic solution = \( \left( 0, -\frac{36}{7}, \frac{2}{7}, 0 \right) \).
\[
\begin{align*}
\mathbf{c}_6 &= \frac{1}{4} \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} \frac{2}{7} \\ \frac{36}{7} \end{pmatrix} \\
\Rightarrow \text{Basic solution} &= (0, 0, \frac{2}{7}, \frac{36}{7})
\end{align*}
\]

9.

The line given by the objective function for \( z = 4 \), when moved parallel to itself towards the origin (as this is a minimization problem), coincides with the boundary line \( n_1 + n_2 = 2 \) of the feasible region and thus the problem has an infinite number of solutions (optimal) as any point on the line segment joining the corners intercepted by \( n_1 + n_2 = 2, \ n_1 = 0, \ n_2 = 0 \) is an optimal solution.

\[ \therefore \ z_{\text{min}} = 2 \] but the number of solutions is infinite.
10. Let \( x_1 \) and \( x_2 \) production runs be undertaken for the processes 1 and 2 respectively, so that our aim is to

Maximize \[ z = 300x_1 + 400x_2 \]

Subject to:

\[ 5x_1 + 4x_2 \leq 200 \]
\[ 3x_1 + 5x_2 \leq 150 \]
\[ 5x_1 + 4x_2 \geq 100 \]
\[ 8x_1 + 4x_2 \geq 80 \]
\[ x_1, x_2 \geq 0 \]

**Graphical Solution**

The line given by the objective function for \( z = 12000 \) when moved parallel to itself away from the origin (since it is the case of maximization) finally encounters the point \( \left( \frac{400}{13}, \frac{150}{13} \right) \) in the feasible region. This is the maximizing point.

\[ \therefore Z_{\text{max}} = 300 \times \frac{400}{13} + 400 \times \frac{150}{13} = \frac{180000}{13} \]
The optimal solution is \( x_1 = \frac{400}{13}, \quad x_2 = \frac{150}{13} \).

Maximum profit = Rs. \( \frac{180000}{13} \).

11. **Graphical Solution**:

Therefore, there is no feasible solution set defined by the constraints.

⇒ The given LPP has no feasible solution.

12. Let \( x_1 \) and \( x_2 \) tons of these two mixtures be produced.

We formulate the given problem mathematically as

\[
\begin{align*}
\text{Max} \quad & z = 15x_1 + 12x_2 \\
\text{S. to} \quad & 6x_1 + 5x_2 \leq 3600 \\
& 3x_1 + 5x_2 \leq 2500 \\
& 8x_1 + 5x_2 \leq 4400 \\
& x_1, x_2 \geq 0,
\end{align*}
\]
The feasible region is given by the shaded area in the figure below.

In this case, the feasible region is not a closed polygon. The dotted line representing the objective function is moved away towards the origin parallel to itself (as it is a minimization problem). It is seen that there is no finite minimum value of \( z \) within the feasible region.

In that case, the problem is said to have an unbounded solution.