Week 2: Assignment

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Note: Follow the notations used in the lectures. Symbols have their usual meanings.

1) The stream function \( \psi(r, \theta) \) in spherical polar coordinate system is given by

(a) \( u_r = \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r} \), \( u_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \theta} \)

(b) \( u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial r} \), \( u_\theta = -\frac{r}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} \)

(c) \( u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial r} \), \( u_\theta = -\frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} \)

(d) \( u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial r} \), \( u_\theta = -\frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} \)

No, the answer is incorrect.
Score: 0

Accepted Answers:
(c) \( u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial r} \), \( u_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \theta} \)

2) The reduced form of the steady state Navier-Stokes equation (2D Cartesian) in terms of stream function is given by

(a) \( \nabla^4 \psi - \nabla^2 \psi = \frac{\partial^2 (\nabla^2 \psi)}{\partial x^2} \)

(b) \( \nabla^4 \psi - \nabla^2 \psi = \frac{\partial^2 (\nabla^2 \psi)}{\partial y^2} \)

(c) \( \nabla^4 \psi = \frac{\partial^2 (\nabla^2 \psi)}{\partial x^2} \)

(d) \( \nabla^4 \psi - \frac{\partial^2 (\nabla^2 \psi)}{\partial x \partial y} = \frac{\partial^2 (\nabla^2 \psi)}{\partial y^2} \)

No, the answer is incorrect.
Score: 0

Accepted Answers:
(c) \( \nabla^4 \psi = \frac{\partial^2 (\nabla^2 \psi)}{\partial x^2} \)

3) If the streamlines corresponding to a two-dimensional flow are represented by \( \psi = x^2 - y^2 \), then the resultant velocity direction at \((2, 2)\) respectively are

(a) \( 2\sqrt{2} \), making an angle 60° to \( x \) axis

(b) \( 2\sqrt{2} \), making an angle 45° to \( x \) axis

(c) \( 4\sqrt{2} \), making an angle 45° to \( x \) axis

(d) \( 4\sqrt{2} \), making an angle 60° to \( x \) axis

No, the answer is incorrect.
Score: 0

Accepted Answers:
(c) \(4\sqrt{2}\), making an angle 45° to \(x\) – axis

4) Let a solid sphere of radius \(a\) be held fixed in a uniform stream \(U\) flowing steadily in the positive \(x\) – direction. Let the fluid be incompressible viscous with viscosity \(\mu\). The corresponding Stokes drag when (in SI units) \(a = 2\) m, \(\mu = 2\) Ns/m², \(U = 7\) m/s is given by

(a) 528 N
(b) 520 N
(c) 428 N
(d) 628 N

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) 528 N

5) Consider the following governing equations corresponding to a two-dimensional viscous incompressible flow in polar coordinates

\[
p\frac{\partial v_r}{\partial r} = -\frac{\partial p}{\partial r} + \mu \left( \frac{1}{r} \frac{\partial}{\partial \theta} \left( r \frac{\partial v_\theta}{\partial \theta} \right) - \frac{v_r}{r} \right)
\]

If the radial velocity is given by \(v_r = \frac{\mu}{\rho} \frac{1}{r} f(\theta)\) for any arbitrary function \(f\) that depends only \(\theta\), then the differential equation satisfied by \(f\) is

(a) \(f'''(\theta) + 2f(\theta)f''(\theta) + 4f'(\theta) = 0\)
(b) \(f'''(\theta) + 2f(\theta)f''(\theta) + 4f'(\theta) = 0\)
(c) \(f'(\theta)^2 + 2f(\theta)f''(\theta) + 4f'(\theta) = 0\)
(d) \(f'''(\theta) + 2f(\theta)f''(\theta) + 4f'(\theta) = 0\)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) \(f'''(\theta) + 2f(\theta)f''(\theta) + 4f'(\theta) = 0\)

6) If \(\vec{u}, \vec{P}\) and \(\Omega\) represent the velocity, pressure and vorticity respectively of a steady state Stokes flow, then the following relations hold

(a) \(\nabla^2 \vec{u} = 0, \nabla^2 \vec{P} = 0, \nabla^2 \vec{\Omega} = 0\)
(b) \(\nabla^4 \vec{u} = 0, \nabla^2 \vec{P} = 0, \nabla^2 \vec{\Omega} = 0\)
(c) \(\nabla^2 \vec{u} = 0, \nabla^2 \vec{P} = 0, \nabla^4 \vec{\Omega} = 0\)
(d) \(\nabla^4 \vec{u} = 0, \nabla^2 \vec{P} = 0, \nabla^4 \vec{\Omega} = 0\)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) \(\nabla^4 \vec{u} = 0, \nabla^2 \vec{P} = 0, \nabla^4 \vec{\Omega} = 0\)

7) For a specific geometry, let us consider the following simplified Navier-Stokes equations

\[
0 = -\frac{\partial p}{\partial r} \quad 0 = -\frac{\partial p}{\partial z} - \rho g
\]

Integrate these to obtain the corresponding pressure. If this pressure satisfies \(p(r, z) = 0\) at \((0, h)\) then we have

(a) \(p = \frac{\mu u z}{2} - \rho g z + \rho gh\)
(b) \(p = \frac{\mu u z}{2} - \rho g z + \rho gh\)
(c) \(p = \frac{\mu u z}{2} - \rho g z + \rho gh\)
(d) \(p = \frac{\mu u z}{2} + \rho g z - \rho gh\)
No, the answer is incorrect.
Score: 0
Accepted Answers:
(a) $p = \frac{\mu u'y'}{2} - \rho g z + \rho g h$

8) The velocity components in case of an axi-symmetric flow are given in $(r, \theta, z)$ cylindrical co-ordinates as $u_z = -\frac{r \psi}{\epsilon}$, $u_r = 0$. Then, the corresponding stream function is given by

(a) $\psi = -\frac{r^2 \psi}{6}$
(b) $\psi = -\frac{r^3 \psi}{6}$
(c) $\psi = -\frac{r^3 \psi}{4}$
(d) $\psi = -\frac{r^4 \psi}{4}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(c) $\psi = -\frac{r^3 \psi}{6}$

9) Consider a two dimensional flow through a narrow passage of length $L$ and width $H$, where $\epsilon = H/L << 1$.

In order to non-dimensionalize the Navier-Stokes equations governing the flow inside the passage, the following dimensionless variables are used: $x' = \frac{x}{L}$, $y' = \frac{y}{L}$, $u' = \frac{u}{U_L}$, $v' = \frac{v}{U_L}$, $p' = \frac{p}{\rho U_L^2}$.

If the corresponding $x$-momentum equation is given by

$$a \left( \frac{\partial u'}{\partial x'} + u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} \right) = -\frac{1}{\alpha} \frac{\partial p'}{\partial x'} + \beta \frac{\partial^2 u'}{\partial x'^2} + \gamma \frac{\partial^2 u'}{\partial y'^2},$$

then the parameters $\alpha, \beta, \gamma$ and $\Lambda$ are given by

(a) $\alpha = \epsilon Re, \beta = \epsilon^2, \gamma = 1, \Lambda = \frac{\mu}{\epsilon^5 P_L}$
(b) $\alpha = \epsilon^2 Re, \beta = \epsilon^2, \gamma = 1, \Lambda = \frac{\mu}{\epsilon^3 P_L}$
(c) $\alpha = \epsilon^2 Re, \beta = \epsilon^2, \gamma = \epsilon^2, \Lambda = \frac{\mu}{\epsilon P_L}$
(d) $\alpha = \epsilon^2 Re, \beta = \epsilon^2, \gamma = 1, \Lambda = \frac{\mu}{\epsilon P_L}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $\alpha = \epsilon^2 Re, \beta = \epsilon^2, \gamma = 1, \Lambda = \frac{\mu}{\epsilon P_L}$

10) Consider an incompressible Newtonian fluid between two parallel disks of radius $R$, separated by a distance $H > \gg R$.

The upper disk is subjected to a velocity $V'$ in $z$-direction and the lower disk is stationary. If the flow is assumed to be axi-symmetric, then the following holds

(a) $v_z$ is substantial but $\frac{\partial v_z}{\partial x}$ is negligible
(b) $v_r$ is substantial but $\frac{\partial v_r}{\partial r}$ is negligible
(c) $v_r$ is substantial and $\frac{\partial v_r}{\partial r}$ is substantial
(d) $v_z$ is substantial but $\frac{\partial v_z}{\partial x}$ is negligible

No, the answer is incorrect.
Score: 0
Accepted Answers:
(b) $v_r$ is substantial but $\frac{\partial v_r}{\partial r}$ is negligible