Assignment of week 2.

① (A)

Supremum means least upper bound. Clearly 7 is an upper bound of the set A because for every $x \in A$, $x < 7$ holds. Now suppose $y = \sup A$ such that $y < 7$. Then by density property of Real numbers there exists $z \in \mathbb{R}$ such that $y < z < 7$. This implies that $z \in A$ and y is not a upper bound. Which is a contradiction. So no real number less than 7 can not be the supremum of A.

So $\sup A = 7$.

② (C).

$$A = \left\{ \frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N} \right\}$$

$$= \left\{ 1+1,\ 1+\frac{1}{2},\ 1+\frac{1}{3},\ \dots,\ \frac{1}{2}+1,\ \frac{1}{2}+\frac{1}{2},\ \frac{1}{2}+\frac{1}{3},\ \dots \right.$$
$$\left. \dots\ \frac{1}{3}+1,\ \frac{1}{3}+\frac{1}{2},\ \frac{1}{3}+\frac{1}{3},\ \dots \right\}$$

Clearly 2 is an upper bound of the set. By the same argument as previous problem it can be shown that $\sup A = 2$.

(3) (B).

$$A = \left\{ \frac{n + (-1)^n}{n} : n \in \mathbb{N} \right\}$$

$$= \left\{ 0, \frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \frac{4}{5}, \cdots \right\}.$$

Now 0 is the lower bound of A. If possible, Let x is the infimum of A where $x > 0$. Then x cannot be the lower bound of A. ~~So 0 is the g~~. So no element $x > 0$ can be an lower bound of A. So $\inf A = 0$.

(4) (B)

Let $x \in \mathbb{R}$ be any point. Now we choose $\varepsilon > 0$ in such a way that the deleted neighbourhood of x i.e, $N(x, \varepsilon) \setminus \{x\}$ does not contain any natural number.

This show that no point $x \in \mathbb{R}$ be a limit point of A. So the set of all limit point of A is ϕ.