Solution of Assignment - 1

(1) We may write

$$W = \{ f_k : f_k(x) = k \text{ for all } x \in \mathbb{R} \}$$

Now suppose a function

$g : W \to \mathbb{R}$ defined as

$g(f_k) = k$, then clearly

g is a one-one onto. so,

$$|W| = |\mathbb{R}|$$

(2) To solve this question we will check each option one by one.

(a) $d(-2,-3) = \min(-2,-3)$

$= -3 < 0$

so, it is not a metric

(b) $d(x,y) = |x-y|$ satisfies all the properties to be a metric trivially. So, it is a metric on $\mathbb{R}$.

(c) $d(-1,1) = |(-1)^2 - 1^2| = 0$

but $-1 \neq 1$

so, it is not a metric.

3 $A =]0,1[$

(a) $\overline{A} = [0,1]$

(b) $int(A) =]0,1[$

(c) $int(A) =]0,1[$

(d) $A' = [0,1]$

So, only (c) matches with the options given in the question.

(4) trivial from definitions.

(5) Let S be infinite (~~Refer~~ follow lecture 1 for better explanation)

Consider $F_n = \{T \subseteq S : |T| = n\}$

Since S is infinite, there exists subsets of all finite cardinalities. So, F_n is non-empty.

Now in each F_n, there must exist an element s_n which is not in other F_n. Now the ~~set~~ collection of all these s_n, say S' is such that

$S' \subseteq S$ and S' is countable.

So, every infinite set has a countable subset.

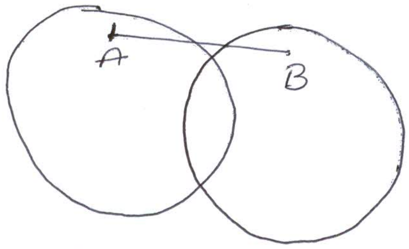
5) (b) the option (b) of (5) is not always true. As for example.

$\mathbb{N}$ = the set of natural no.

is an infinite set but does not have an uncountable subset. because it is itself a countable set.

6

~~(a)~~



clearly the picture of is a union of two convex set. But the line AB does not contained in the union. So, union of convex sets can not be always convex.

again suppose S and T are two convex set.

let $U = S \cap T$

let A and B be any arbitrary points of U

let $W = \{ x \mid x \text{ is a point of the line segment } AB \}$