Assignment – 9

1) Which of the following function is uniformly continuous?
   a) $f(x) = \frac{1}{x}$ on]0, 1]
   b) $f(x) = sin(x^2)$ on $[0, \infty[$
   c) $f(x) = sin(x)$ on $[0, \infty[$
   d) None of the above

Answer: c)

2) Which of the following function is not uniformly continuous?
   a) $f(x) = x^2$ on [0, 1]
   b) $f(x) = x^3$ on [0, 2]
   c) $f(x) = \sqrt{x}$ on [0, 3]
   d) $f(x) = x^2$ on $[0, \infty[$

Answer: d)

3) If f and g are uniformly continuous on the same interval, then
   a) $f + g$ is also a uniformly continuous function on the interval
   b) $f + g$ is not a uniformly continuous function on the interval
   c) $f + g$ is not a continuous function on the interval
   d) None of the above

Answer: a)

4) If a continuous function $f(x)$ has a maximum or minimum value at x_0 and is also differentiable at this point, then

   a) $f'(x_0) \neq 0$
   b) $f'(x_0) > 0$
   c) $f'(x_0) = 0$
   d) $f'(x_0) < 0$

Answer: c)

5) Let f be a continuous real-valued function on [0,1] such that $f(0) = -1$ and $f(1) = 1$, then there always exist a $t \in [0, 1]$ such that

   a) $f(t) = -2$
   b) $f(t) = 2$
   c) $f(t) = 3/2$
   d) $f(t) = -1/2$

Answer: d)

6) The function $f(x)=\begin{cases} x\,sin\dfrac{1}{x},\ x\neq 0 \\ 0,\ x=0 \end{cases}$ is

a) absolutely continuous but not continuous
b) neither continuous nor absolutely continuous
c) continuous and absolutely continuous both
d) continuous but not absolutely continuous

Answer: d)

7) Let $f(x)=\begin{cases} \dfrac{x-|x|}{x}\ when\ x\neq 0 \\ 2\ when\ x=0 \end{cases}$. Then

a) f has removal discontinuity at $x=0$
b) f has discontinuity of first kind at $x=0$
c) f has discontinuity of second kind at $x=0$
d) None of the above

Answer: b)

8) Let $f(x)=\begin{cases} 1,\ when\ x\ is\ irrational \\ -1,\ when\ x\ is\ rational \end{cases}$. Then

a) f is discontinuous at every point of $\mathbb{R}$
b) f is discontinuous at finite number of points of $\mathbb{R}$
c) f is discontinuous at $x=0$ only
d) None of the above

Answer: a)

9) Let $f(x)=\begin{cases} \dfrac{sin2x}{x}\ when\ x\neq 0 \\ 1\ when\ x=0 \end{cases}$. Then

a) f has removal discontinuity at $x=0$
b) f has discontinuity of first kind at $x=0$
c) f has discontinuity of first kind from the right at $x=0$
d) f has discontinuity of second kind at $x=0$

Answer: a)

10) Let $f(x)=\begin{cases} x,\ when\ x\ is\ irrational \\ -x,\ when\ x\ is\ rational \end{cases}$. Then

a) f is continuous at every point of $\mathbb{R}$
b) f is discontinuous at finite number of points of $\mathbb{R}$
c) f is continuous at $x=0$ only
d) None of the above

Answer: c)