Week 8 Assignment

1. \( \lim_{x \to \infty} x \sin \frac{1}{x} \) equals to

   (A) 1
   (B) 0
   (C) -1
   (D) None of these

2. \( f(x) = \left\{ \sin \frac{1}{x} : x \in (0,1) \cup \{2\} \right\} \), then

   (A) \( f(x) \) is continuous
   (B) \( f(x) \) has only one point of discontinuity
   (C) \( f(x) \) has only two point of discontinuity
   (D) None of these

3. Which of the following function is continuous at origin

   (A) \( f(x) = \left\{ \begin{array}{ll} \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{array} \right. \)

   (B) \( f(x) = \left\{ \begin{array}{ll} x + \sin \frac{1}{x}, & x \neq 0 \\ 1, & x = 0. \end{array} \right. \)
(C) \( f(x) = \begin{cases} \sin(x) \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases} \)

(D) \( f(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ 1, & x = 0. \end{cases} \)

4. Let the functions \( f \) and \( g \) are defined on an interval \( I \). If \( f \) and \( g \) are continuous at a point \( p \) in \( I \) then,

(A) \( fg \) may be continuous at \( p \)

(B) \( fg \) discontinuous at \( p \)

(C) \( fg \) continuous at \( p \)

(D) None of the above.

5. The function \( f(x) \) is defined as

\[
f(x) = \begin{cases} 2, & x = 2 \\ 1, & x \neq 2. \end{cases}
\]

then

(A) \( \lim_{x \to \infty} f(x) = 2 \)

(B) \( f(x) \) is continuous

(C) \( \lim_{x \to \infty} f(x) \) does not exist

(D) \( f(x) \) is discontinuous
6. Let the functions $f$ and $g$ are defined on an interval $I$. If $f$ and $g$ are continuous at a point $p$ in $I$ then

(A) $f/g$ is continuous at $p$

(B) $f/g$ is discontinuous at $p$

(C) $f/g$ is continuous at $p$, provided $g(p)$ not equals to 0

(D) None of these

7. The function $f(x) = \begin{cases} \cos x, & x \geq 0 \\ -\cos x, & x < 0. \end{cases}$

(A) Continuous at 0

(B) Discontinuous at 0

(C) Discontinuous for $x<0$

(D) None of these

8. The function $f(x) = \begin{cases} 2x, & 0 \leq x < 1 \\ 3, & x = 1 \\ 4x, & 1 < x \leq 2 \end{cases}$

(A) Continuous at 1

(B) $\lim_{x \to 1} f(x) = 3$

(C) discontinuous at 1

(D) discontinuous on $(0, 1)$
9. The function \( f(x) = \begin{cases} \frac{e^x - x - 1}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases} \)

(A) Continuous at 0
(B) Discontinuous at 0
(C) \( \lim_{x \to 0} f(x) = 1 \)
(D) None of the above

10. \( \lim_{n \to \infty} \sin \left( \pi \sqrt{\frac{1}{n^2} + 1} \right) \)

(A) 0
(B) 1
(C) -1
(D) does not exist.