Assignment-3

(1) In the set of real numbers, the set \((a, b) \subset \mathbb{R}\) is
   (a) Closed.
   (b) Compact.
   (c) Closed but not compact.
   (d) Not compact.

(2) In the set of real numbers (not extended real number system), the set \([0, \infty) \subset \mathbb{R}\) is
   (a) Closed and bounded.
   (b) Closed but not bounded.
   (c) Open and bounded.
   (d) Open but not bounded.

(3) Suppose A is a bounded set in \(\mathbb{R}\), then closure of A
   (a) Closed but not bounded.
   (b) Bounded but not closed.
   (c) Compact.
   (d) Open and bounded.

(4) The sequence \(\{s_n\}\), where \(s_n = 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\)
   is
   (a) Bounded above and increasing.
   (b) Bounded below and decreasing.
   (c) Neither increasing nor decreasing.
   (d) Unbounded.

(5) The derive set of the sequence \(\{s_n\}\), where \(s_n = \sin \frac{n\pi}{2} + \frac{(-1)^n}{n}\), \(n \in \mathbb{Z}\)
   is
   (a) \(\{1, 0\}\).
   (b) \(\{1, 0, -1\}\).
   (c) \(\{-1, 0\}\).
   (d) \(\{1, -1\}\).

(6) The sequence \(\{s_n\}\), where \(s_n = 1 + (-1)^n\), \(n \in \mathbb{Z}\)
   is
   (a) Periodic with period 1.
   (b) Periodic with period 2.
   (c) Not periodic.
   (d) Convergent.
(7) \[ \lim_{n \to \infty} \frac{3 + 2\sqrt{n}}{\sqrt{n}} \]

(a) 1.
(b) 2.
(c) 3.
(d) 5.

(8) If \( \{a_n\} \) is a convergent sequence of real numbers satisfying \( a_{n+1}^2 = 2a_n - 1 \), then

(a) \( \lim_{n \to \infty} a_n = 2 \).
(b) \( \lim_{n \to \infty} a_n = 1 \).
(c) \( \lim_{n \to \infty} a_n = -1 \).
(d) \( \lim_{n \to \infty} a_n = -2 \).

(9) If \( a > 0 \), then \( \lim_{n \to \infty} \sqrt[n]{a} \) is

(a) 1.
(b) 2.
(c) \( \sqrt{2} \).
(d) 0.

(10) \( \lim_{n \to \infty} \sqrt[n]{n} \) is

(a) 1.
(b) 2.
(c) Diverges to infinity
(d) 0.

Answers
1. (d)
2. (b)
3. (c)
4. (a)
5. (b)
6. (b)
7. (b)
8. (b)
9. (a)
10. (a)