Assignment - 12

1) If $f(x)=x^2$ for rational values of x in the interval $(0,2)$ and $f(x)=x^3$ for irrational values of x in the same interval, and let upper R-integral i.e. $\overline{\int_a^b} f\ dx$ be U and lower R-integral i.e. $\underline{\int_a^b} f\ dx$ be L. Then

a) $L=0$, $U=2$

b) $L=2\frac{7}{12}$, $U=4\frac{1}{12}$

c) $L=U=4$

d) None of the above

Ans: b)

2) Let c be a fixed point in [0, 1]. A function $f:[0,1]\to\mathbb{R}$ is defined by $f(x)=\begin{cases} c, if\ 0\le x\le c \\ 2c, if\ c<x\le 1 \end{cases}$.

It is given that the Riemann integral $\int_0^1 f(x)dx$ is $\frac{7}{16}$. Then, the value of c is

a) $\frac{1}{2}$,

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) $\frac{1}{5}$

Ans: c)

3) Let $f(x)=\sin x$, $\alpha(x)=x^2$ and $\beta(x)=x^3$ on $[0,\frac{\pi}{2}]$. Then

a) $f\in R(\alpha)$ and $f\in R(\beta)$ both

b) $f\in R(\alpha)$ but $f\notin R(\beta)$

c) $f\in R(\beta)$ but $f\notin R(\alpha)$

d) Neither $f\in R(\alpha)$ nor $f\in R(\beta)$

Ans: a)

4) If $f\in R(\alpha)$ on [a , b], then

a) $|f| \in R(\alpha)$ and $\left|\int_a^b f(x)d\alpha(x)\right| \geq \int_a^b |f(x)|\, d\alpha(x)$

b) $|f| \notin R(\alpha)$ and $\left|\int_a^b f(x)d\alpha(x)\right| \leq \int_a^b |f(x)|\, d\alpha(x)$

c) $|f| \in R(\alpha)$ and $\left|\int_a^b f(x)d\alpha(x)\right| \leq \int_a^b |f(x)|\, d\alpha(x)$

d) None of the above

Ans: c)

5) Let α be a monotonically increasing function on $[a, b]$ such that α' belongs to the class of Riemann integrable function on $[a, b]$ and let f be a bounded real function on $[a, b]$. Then

a) $\int_a^b f d\alpha = \int_a^b f'(x)\alpha(x)dx$

b) $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$

c) $\int_a^b f d\alpha = \int_a^b f'(x)\alpha'(x)dx$

d) None of the above

Ans: b)

6) Let $f(x) = x^2$ and $\alpha(x) = x^2$ on [0, 2]. Then the Riemann – Stieltjes integral $\int_0^2 f d\alpha =$

a) 8
b) 32/5
c) 2
d) None of the above

Ans: a)

7) Let $f(x) = [x]$, and $\alpha(x) = x^2$ on [0, 2], where [ ] represents the greatest integer function. Then the Riemann – Stieltjes integral $\int_0^2 f d\alpha =$

a) 4
b) 16/3
c) 3
d) None of the above

Ans: c)

8) Which of the following function on [0, 1] is not a Riemann integrable function?

a) $f(x) = \dfrac{1}{x+1}$

b) $f(x)=\begin{cases} x\cos\dfrac{1}{x}, \ x\neq 0 \\ 0, \ x=0 \end{cases}$

c) $f(x)=\begin{cases} \sin\dfrac{1}{x}, \ x \text{ is irrational} \\ 0, otherwise \end{cases}$

d) None of the above

Ans: c)

9) Which of the following function is not a Riemann integrable function on [0, 1]?

a) $f(x)=\begin{cases} \dfrac{1}{x-1}, \ x\neq 1 \\ 0, \ x=1 \end{cases}$

b) $f(x)=\begin{cases} 2n, \ if \ \ x=\dfrac{1}{n} \ where \ n=1,2,... \\ 0, \ otherwise \end{cases}$

c) $f(x)=x^2$

d) None of the above

Ans: b)

10) A function f is defined on [0, 1] as follows:

$f(x)=\begin{cases} \dfrac{1}{a^{r-1}}, \ when \ \dfrac{1}{a^r}<x\leq\dfrac{1}{a^{r-1}} \ \ for \ r=1,2,3,... \\ 0, \ when \ x=0 \end{cases}$, where a is an integer greater than 2.

Then the Riemann integral $\int_0^1 f\,dx=$

a) $\dfrac{a}{a+1}$

b) $\dfrac{a+1}{a}$

c) $\dfrac{a-1}{a}$

d) None of the above

Ans: a)