Week 11 Assignment

1. Which of the following ensures that the equation $a_0x^n + a_1x^{n-1} + \cdots + a_n = 0$ has at least one root between 0 and 1?

(A) $na_0 + (n-1)a_1 + \cdots + a_{n-1} = 0$

(B) $\frac{a_0}{n+1} + \frac{a_1}{n} + \cdots + a_n = 0$

(C) $a_0 + a_1 + \cdots + a_n = 0$

(D) None of these

2. If a tangent to the curve $f(x) = x^2$ at a point (c, f(c)) is parallel to the line joining (a, f(a)) and (b, f(b)) then,

(A) a, c, b are in GP

(B) a, c, b are in AP

(C) a, c, b are in HP

(D) None of these

3. If $f(x) = x^5 - x - 1,\ x \in [1, 2]$ then which of the following is true?

(A) f(x) is strictly increasing in [1, 2]

(B) f(x) is decreasing in [1, 2]

(C) f(x) is strictly decreasing in [1, 2]

(D) None of the above

4. $\lim_{x\to -4}\frac{\sin(\pi x)}{x^2-16}$ is

(A) $-\frac{\pi}{8}$

(B) $\frac{\pi}{8}$

(C) 0

(D) None of the above.

5. $\lim_{x\to 0}\frac{\sin(2x)+7x^2-2x}{x^2(x+1)^2}$ is

(A) 7

(B) -7

(C) 0

(D) None of the above

6. $\lim_{x\to\infty}(e^x+x)^{\frac{1}{x}}$ is

(A) 0

(B) 1

(C) e

(D) None of these

7. Let f(x) be a continuous function on the interval [0, 2] and f(0) = 1, f(1) = k and f(2) = 2. Then which of the following ensures that f(x) = 1/2 has at least two solution in [0, 2]?

(A) k = 1

(B) k = 0

(C) k = 2

(D) None of these

8. If sin x has a Taylor series expansion about $\frac{\pi}{2}$ of the form

$a_0 + a_1\left(x - \frac{\pi}{2}\right) + a_2\left(x - \frac{\pi}{2}\right)^2 + \cdots$, then the values of a_3 and a_4 are respectively

(A) 0, 1/24

(B) 0, -1/24

(C) 1, 1/24

(D) None of the above

9. Let f be a function defined on [0, 1] as $f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational.} \end{cases}$

Then the values of L(P, f) and U(P, f) for any partition P of [0, 1]

(A) L(P, f) = 1, U(P, f) = 0

(B) L(P, f) = 0, U(P, f) = 1

(C) L(P, f) = 0, U(P, f) = 2

(D) None of the above

10. If f is a real valued function on [a, b] and m and M are greatest lower bound and least upper bound of f respectively, then

(A) m (b - a) = M (b - a)

(B) $m(b-a) \geq M(b-a)$

(C) $m(b-a) \leq M(b-a)$

(D) None of these