Week 11 Assignment

1. Which of the following ensures that the equation \(a_0x^n + a_1x^{n-1} + \cdots + a_n = 0\) has at least one root between 0 and 1?

   (A) \(na_0 + (n-1)a_1 + \cdots + a_{n-1} = 0\)

   (B) \(\frac{a_0}{n+1} + \frac{a_1}{n} + \cdots + a_n = 0\)

   (C) \(a_0 + a_1 + \cdots + a_n = 0\)

   (D) None of these

2. If a tangent to the curve \(f(x) = x^2\) at a point \((c, f(c))\) is parallel to the line joining \((a, f(a))\) and \((b, f(b))\) then,  

   (A) \(a, c, b\) are in GP

   (B) \(a, c, b\) are in AP

   (C) \(a, c, b\) are in HP

   (D) None of these

3. If \(f(x) = x^5 - x - 1, x \in [1, 2]\) then which of the following is true?

   (A) \(f(x)\) is strictly increasing in [1, 2]

   (B) \(f(x)\) is decreasing in [1, 2]

   (C) \(f(x)\) is strictly decreasing in [1, 2]

   (D) None of the above
4. \( \lim_{x \to 4} \frac{\sin(\pi x)}{x^2 - 16} \) is

(A) \(-\frac{\pi}{8}\)

(B) \(\frac{\pi}{8}\)

(C) 0

(D) None of the above.

5. \( \lim_{x \to 0} \frac{\sin(2x) + 7x^2 - 2x}{x^2 (x+1)^2} \) is

(A) 7

(B) -7

(C) 0

(D) None of the above

6. \( \lim_{x \to \infty} \left( e^x + x \right)^{\frac{1}{x}} \) is

(A) 0

(B) 1

(C) e

(D) None of these
7. Let \( f(x) \) be a continuous function on the interval \([0, 2]\) and \( f(0) = 1 \), \( f(1) = k \) and \( f(2) = 2 \). Then which of the following ensures that \( f(x) = 1/2 \) has at least two solution in \([0, 2]\)?

(A) \( k = 1 \)
(B) \( k = 0 \)
(C) \( k = 2 \)
(D) None of these

8. If \( \sin x \) has a Taylor series expansion about \( \frac{\pi}{2} \) of the form

\[
a_0 + a_1 \left(x - \frac{\pi}{2}\right) + a_2 \left(x - \frac{\pi}{2}\right)^2 + \cdots,
\]

then the values of \( a_3 \) and \( a_4 \) are respectively

(A) 0, 1/24
(B) 0, -1/24
(C) 1, 1/24
(D) None of the above

9. Let \( f \) be a function defined on \([0, 1]\) as

\[
f(x) = \begin{cases} 1, & \text{x is rational} \\ 0, & \text{x is irrational} \end{cases}
\]

Then the values of \( L(P, f) \) and \( U(P, f) \) for any partition \( P \) of \([0, 1]\)

(A) \( L(P, f) = 1, U(P, f) = 0 \)
(B) \( L(P, f) = 0, U(P, f) = 1 \)
(C) \( L(P, f) = 0, U(P, f) = 2 \)
(D) None of the above
10. If \( f \) is a real valued function on \([a, b]\) and \( m \) and \( M \) are greatest lower bound and least upper bound of \( f \) respectively, then

(A) \( m (b - a) = M (b - a) \)

(B) \( m(b - a) \geq M (b - a) \)

(C) \( m(b - a) \leq M (b - a) \)

(D) None of these