Assignment-07

The due date for submitting this assignment has passed.

As per our records you have not submitted this assignment.

Due on 2021-03-10, 23:59 IST.

1) Consider the equation $u_t = Ku_{xx}$, $x \in \mathbb{R}$. Then

- $(x, t) \mapsto u(x, t)$ is a solution if $u$ is any solution and $y \in \mathbb{R}$ is fixed.
- $u_x, u_t, u_{xx}$ are solutions if $u$ is a solution.
- $au + \beta v$ is a solution if $a, \beta \in \mathbb{R}$ and $u, v$ are solutions.

None of the above.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$u(x, t) \equiv c$ for any fixed solution $c$.

2) Let $u(x, t) = \mathcal{S}(t \phi(x))$ for some solution operator $\mathcal{S}$ including $u_t - K u_{xx} = f(x, t)$, $t \geq 0$; $u(x, 0) = \tilde{u}(x)$. Then

- $u(x, t) = \mathcal{S}(\phi(x) + \int_0^t \mathcal{S}(t-s)f(x,s)ds)$ solves $u_t - Ku_{xx} = f(x, t)$, $t \geq 0$; $u(x, 0) = \tilde{u}(x)$.
- $u(x, t) = -\mathcal{S}(\phi(x) + \int_0^t \mathcal{S}(t-s)f(x,s)ds)$ solves $u_t - Ku_{xx} = -f(x, t)$, $t \geq 0$; $u(x, 0) = \tilde{u}(x)$.
- $u_t = \mathcal{S}(\phi(x) + \int_0^t \mathcal{S}(t-s)f(x,s)ds)$ solves $u_t - Ku_{xx} = f(x, t)$, $t \geq 0$; $u(x, 0) = \tilde{u}(x)$.

None of the above.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$u(x, t) = \mathcal{S}(\phi(x) + \int_0^t \mathcal{S}(t-s)f(x,s)ds)$ solves $u_t - Ku_{xx} = f(x, t)$, $t \geq 0$; $u(x, 0) = \tilde{u}(x)$.

3) Let $f \in C^1([0, \infty) \times \mathbb{R})$ and $\Phi$ be the fundamental solution of the heat equation. Then

- $\int_0^t \Phi(\sqrt{\alpha}, s)f(x - \sqrt{\alpha}y, s)ds = f(x, t)$.

None of the above.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$f(x, t)$ solves the heat equation $u_t - Ku_{xx} = f(x, t)$, $t > 0$;

$f(x, y)$ is defined for $x, y \in \mathbb{R}$ and $t > 0$.

4) Let $u_1$ be the solution of $u_1 - \Delta u_1 = f$ in $\mathbb{R}^n \times (0, \infty)$; $u_1 |_{t=0} = 0$ on $\mathbb{R}^n \times \{0\}$ and $u_2$ be any solution of $u_2 - \Delta u_2 = g$ in $\mathbb{R}^n \times (0, \infty)$; $u_2 |_{t=0} = 0$ on $\mathbb{R}^n \times \{0\}$. Then

- $u_1 - u_2$ solves $u_1 - \Delta u_1 = f$ in $\mathbb{R}^n \times (0, \infty)$; $u_1 |_{t=0} = 0$ on $\mathbb{R}^n \times \{0\}$.
- $u_1 + u_2$ solves $u_2 - \Delta u_2 = g$ in $\mathbb{R}^n \times (0, \infty)$; $u_2 |_{t=0} = 0$ on $\mathbb{R}^n \times \{0\}$.
- $-u_1 + u_2$ solves $u_1 - \Delta u_1 = -f$ in $\mathbb{R}^n \times (0, \infty)$; $u_1 |_{t=0} = 0$ on $\mathbb{R}^n \times \{0\}$.

None of the above.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$u_1 + u_2$ solves $u_1 - \Delta u_1 = f$ in $\mathbb{R}^n \times (0, \infty)$; $u_1 |_{t=0} = 0$ on $\mathbb{R}^n \times \{0\}$.

5) Let $\Phi$ be the fundamental solution of the heat equation, $f \in C([0, \infty) \times \mathbb{R})$ have compact support, and $u(x, t) = \int_0^t \int_{\mathbb{R}} \Phi(x, y, s)f(x-y, t-s)dyds$. Then

- $u_{x_k}(x, t) = \int_0^t \int_{\mathbb{R}} \Phi(x, y, s)f_{x_k}(x-y, t-s)dyds$, $1 \leq k \leq n$.
- $u_{y_i}(x, t) = \int_0^t \int_{\mathbb{R}} \Phi(x, y, s)f_{y_i}(x-y, t-s)dyds + \int_0^t \Phi(x, y, s)f_x(x-y, 0)dy$, $1 \leq i \leq n$.
- $u_{y_i}(x, t) = \int_0^t \int_{\mathbb{R}} \Phi(x, y, s)f_{y_i}(x-y, t-s)dyds + \int_0^t \Phi(x, y, s)f_y(x-y, 0)dy$.
- $u_y(x, t) = \int_0^t \int_{\mathbb{R}} \Phi(x, y, s)f_y(x-y, t-s)dyds$.

None of the above.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$u_{x_k}(x, t) = \int_0^t \int_{\mathbb{R}} \Phi(x, y, s)f_{x_k}(x-y, t-s)dyds$, $1 \leq k \leq n$.

$w(x, t) = \int_0^t \int_{\mathbb{R}} \Phi(x, y, s)f_{x_k}(x-y, t-s)dyds + \int_0^t \Phi(x, y, s)f_y(x-y, 0)dy$. 

$w(x, t) = \int_0^t \int_{\mathbb{R}} \Phi(x, y, s)f_{y_i}(x-y, t-s)dyds$